Week 3 (due April 20)

1. A model with a moduli space of supersymmetric vacua is described at low energies by a supersymmetric version of the sigma-model, whose target is the space of vacua. That is, the low-energy effective action has the form

\[ S = -\frac{1}{2} \int d^4x \, g_{ij}(\phi) \partial_\mu \phi^i(x) \partial^\mu \phi^j(x) + \ldots, \]

where dots denote terms containing fermions, and \( g_{ij}(\phi) \) is a Riemannian metric on the target space with coordinates \( \phi^i \).

(a) Consider \( N = 1 \) SUSY gauge theory with gauge group \( U(1) \) and two chiral superfields of charge +1. Determine \( g_{ij}(\phi) \) as a function of the FI parameter. (The target space here is \( S^2 \) if \( \zeta > 0 \), a single point if \( \zeta = 0 \), and empty if \( \zeta < 0 \).)

(b) The same, but for two chiral superfields of charges \( Q_1 \) and \( Q_2 \). Consider separately the cases when \( Q_1 \) and \( Q_2 \) have the same sign and when they have opposite signs.