These are notes on accuracy, experimental error, and error analysis.

I. WHAT IS ACCURACY?

The 'accuracy' of an experimentally measured value is related to the possible difference between the measured value and the actual or true value\(^1\). The accuracy of a measurement is determined entirely by its experimental uncertainty. Note that theoretical predictions have not been mentioned in this discussion. A common misunderstanding is that the accuracy of a measurement is somehow related to the discrepancy between the measurement and theoretical predictions. This is not a measure of accuracy; it is a measure of the correctness and/or appropriateness of the theory as applied to the experiment. This confusion likely arises because the theory you learn in Ph1 is already well established as a successful model for the behavior of your experiments. Thus discrepancies between that theory and your experimental results suggest experimental errors (inaccuracy, as described above) instead of incorrect or inapplicable theory. Don't let this confuse you: if the uncertainties are calculated correctly a measurement of 2.35±0.03V is always more accurate than a measurement of 2.38±0.08V, regardless of whether the 'theory' predicts 2.38 or 2.34V.

II. WHAT IS EXPERIMENTAL ERROR?

Experimental errors arise from both systematic flaws in measurement equipment and the resolution of your physical senses\(^2\). The 5% error in your meter is due to systematic flaws in its construction which result in the possibility of it measuring a quantity as much as 5% off from its actual value. The term 'systematic' refers to the idea that these errors are the same for every measurement and thus highly correlated with each other\(^3\) (though only for a particular setting on a particular meter!).

Along with the systematic error in your meter, there is measurement error due to your senses. This is simply the fact that you cannot see the difference between, say, 3.733V and 3.734V on your meter using 5V full scale. Unlike systematic errors, these errors are not the same for every measurement (for example, with one measurement you may read the needle slightly higher than it actually is, while for another you may read it slightly lower).

Suppose a quantity depends on several measured parameters: \( S \rightarrow S(A, B, \ldots) \) where \( A, B \), etc. are measured and \( S \) is a function of all of them. If the errors in \( A, B \), etc. are uncorrelated, then some of these errors will conspire to increase \( S \) and some will conspire to decrease \( S \), such that in general the error in \( S \) will be 'less than the sum of the error in its parts.' Correlated systematic errors will all conspire to push \( S \) in the same direction, and therefore these errors sum as one would naively expect.

A quantitative example may clarify this. Suppose \( S \rightarrow S_a \) is simply the average value of some voltage that we measure twice, obtaining \( A \) and \( B \). Clearly

\[
S_a = \frac{A + B}{2}
\]  

\(^1\) This presumes that a 'true' value exists, i.e. that nature has objective properties independent of our skill at measuring them. This is the cornerstone of scientific thought.

\(^2\) Digital equipment also carries resolution limitations. Also, sometimes some forms of theoretical errors are included in experimental measurements; this is essentially the case with statistical uncertainties.

\(^3\) From this description it is clear that voltage changes on a particular meter setting are actually measured more accurately than 5%, since some systematic errors will redundantly cancel in the difference. We do not consider this effect in Ph1.
If \( A \) has some uncertainty \( \pm \delta A \) and \( B \) has some uncertainty \( \pm \delta B \), then \( S_a \) also has some uncertainty, \( \pm \delta S_a \). If the errors in \( A \) and \( B \) are correlated then they will all add (or subtract) together in \( S_a \) and the uncertainty \( \delta S_a \) will be

\[
\delta S_a = \frac{\delta A + \delta B}{2}
\]  

(2)

It should be clear we haven’t gained any accuracy by averaging. A physical analogy involves a ruler with millimeter graduations that are slightly too long. When you measure some length with this ruler, you will always measure a length that is shorter than the actual length. Performing multiple measurements and averaging does not change this fact.

The situation is different if the errors \( \delta A \) and \( \delta B \) are uncorrelated. Then one may push \( S \) to be larger while the other pushes \( S \) to be smaller, with a net cancellation effect\(^4\). Therefore, uncorrelated errors typically ‘sum’ to less than correlated errors. A physical analogy involves using many rulers, some with millimeter markers that are too long and others that are too short. If the error in the meters is random, the more rulers you use and average, the more accurate your measurement.

For uncorrelated errors \( \delta A \) and \( \delta B \) the error \( \delta S \) is usually taken to be

\[
\delta S = \sqrt{\delta S_A^2 + \delta S_B^2}, \quad \text{where} \quad \delta S_A = \frac{dS}{dA} \delta A \quad \text{and} \quad \delta S_B = \frac{dS}{dB} \delta B.
\]  

(4)

Here \( \delta S_A \) is the error in \( S \) due to \( A \), etc. The above derivatives are ‘partial derivatives,’ (usually denoted with a \( \partial \) instead of a \( d \)) which means they are taken keeping all other variables constant. For \( S_a \) given above, \( \delta S_A = \delta A/2 \), etc., so

\[
\delta S_a = \frac{1}{2} \sqrt{\delta A^2 + \delta B^2}
\]  

(5)

which is always less than Eq. (2). Thus, reading errors can be reduced by performing the reading several times and averaging\(^5\).

### III. HOW TO CALCULATE ERRORS IN PH1

For PH1, you are not expected to evaluate your experimental uncertainties to the precision suggested above, however you are expected to understand the sources and approximate sizes of your uncertainties, since they tell you the accuracy of your results. When you have more than

---

\(^4\) If a set of measurements \( A_i \), \( i = 1 \ldots N \), are uncorrelated, then the \( A_i \) form a random distribution about the ‘true’ value of the measurement, \( V \). The average is

\[
\sum_{i=1}^{N} A_i = V + \sum_{i=1}^{N} \frac{A_i - V}{N}
\]

The terms to the right of \( V \) are the average of a series of randomly distributed numbers. A simple computer algorithm can be used to demonstrate such a series grows proportionally to \( \sqrt{N} \), such that its average shrinks as \( \sqrt{N}/N = 1/\sqrt{N} \).

\(^5\) Note that it is possible that the errors \( \delta A \) and \( \delta B \) will each push \( S \) the same way, perhaps to be larger. Then the error in \( S \) will actually be larger than the \( \delta S \) above. Thus the above \( \delta S \) represents a ‘typical’ error and not an absolute limit to the error. If \( \delta A \) and \( \delta B \) represent the standard deviations of Gaussian distributions of errors in \( A \) and \( B \), then \( \delta S \) as calculated above is exactly the standard deviation of a Gaussian distribution of errors in \( S \). Other situations are often well approximated with these ideas.
one source of error, for Ph1 you need keep only the most significant source. This is because if 
\[ \delta S_A \gg \delta S_B \]
\[ \delta S = \sqrt{\delta S_A^2 + \delta S_B^2} \sim \delta S_A \]  
(6)
while if \( \delta S_A \sim \delta S_B \)
\[ \delta S = \sqrt{\delta S_A^2 + \delta S_B^2} \sim \sqrt{2} \delta S_A \sim \delta S_A. \]  
(7)

You need to estimate \( \delta S_A \) and \( \delta S_B \) to know which to keep. This is usually easy. For example, if \( A \) and \( B \) enter \( S \) as a sum, \( S = C(A+B) \) with \( C \) constant, then \( \delta S_A = C \delta A \) and \( \delta S_B = C \delta B \) and the largest absolute error contributes the most. If \( A \) and \( B \) enter as a product or quotient, \( S = CAB \) or \( S = CA/B \), then \( \delta S_A/S = \delta A/A \) and \( \delta S_B/S = \delta B/B \), and the largest percentage error contributes the most. Often, the assumed 5\% error of your meter will be the largest contribution to your errors, and since it is systematic repeated measurements do not reduce that error. However, be careful measuring large resistances and very small voltage changes, as in these circumstances reading error can exceed 5\% note these uncertainties, if not explicitly then by the number of significant digits you keep.