Physics 1b Prac — ANALOG SIGNAL PROCESSING

Capacitors — in particular, $RC$ circuits — can do calculus. That is, for a given input voltage signal $V(t)$, viewed as a function of time, the simplest $RC$ circuit can be constructed such that the output voltage is either the derivative or integral of $V(t)$. The mathematical derivations of these properties are deceptively simple; each of the small number of steps is very simple. (For me, the direction in which each derivation seems to be heading seems odd, and then the final step, albeit trivial, is always something of a surprise.) But the results are of enormous practical utility.

![](image)

Following voltages around the circuit, using the sign conventions implied in the figure, and using Ohm's Law and the definition of $C$, one finds

$$V_{in}(t) - V_R(t) - V_C(t) = 0 \text{ or } V_{in} - IR - \frac{Q}{C} = 0$$

and also

$$I = \frac{dQ}{dt} \text{ or, equivalently, } Q = \int I dt.$$  

So, e.g.,

$$V_R = V_{in} - \frac{Q}{C} \quad V_R = V_{in} - \frac{1}{C} \int dt' \quad V_R = V_{in} - \frac{1}{RC} \int IR dt' \quad V_R = V_{in} - \frac{1}{RC} \int t V_{R} dt'.$$

**IF** $V_R \ll V_{in}$, i.e., if $V_{in}$ is SLOW on the $RC$ time scale and $RC$ can be thought of as a very small time in the above equation, **THEN**