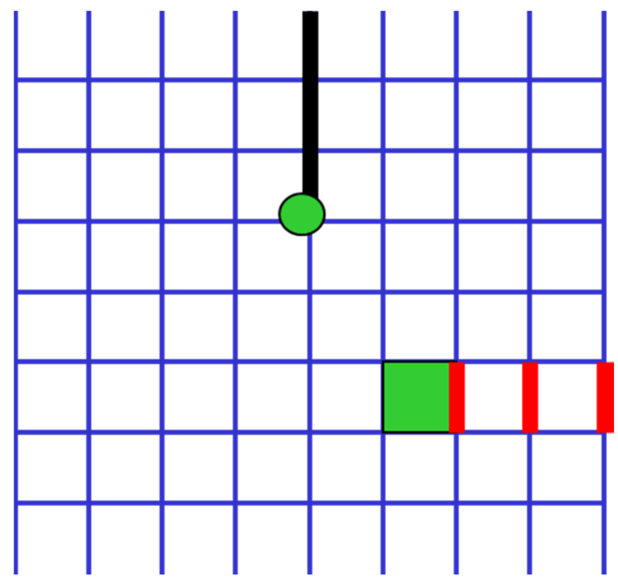
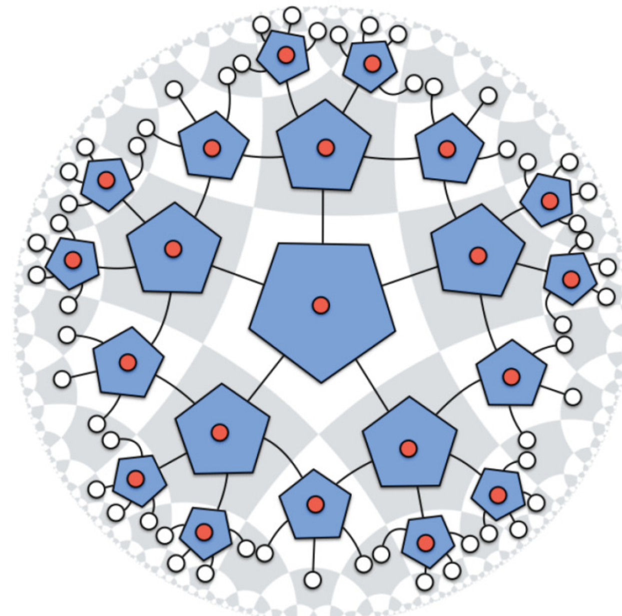
Stability, Topology, Holography:
The many facets of quantum error correction



topological quantum code



holographic quantum code

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

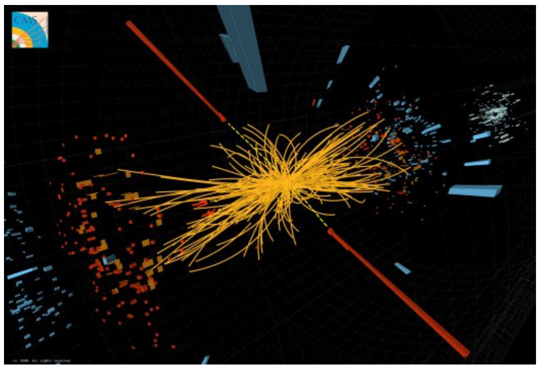
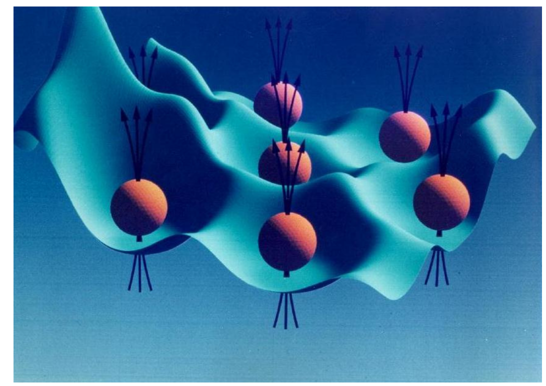
GORDON AND BETTY MOORE FOUNDATION

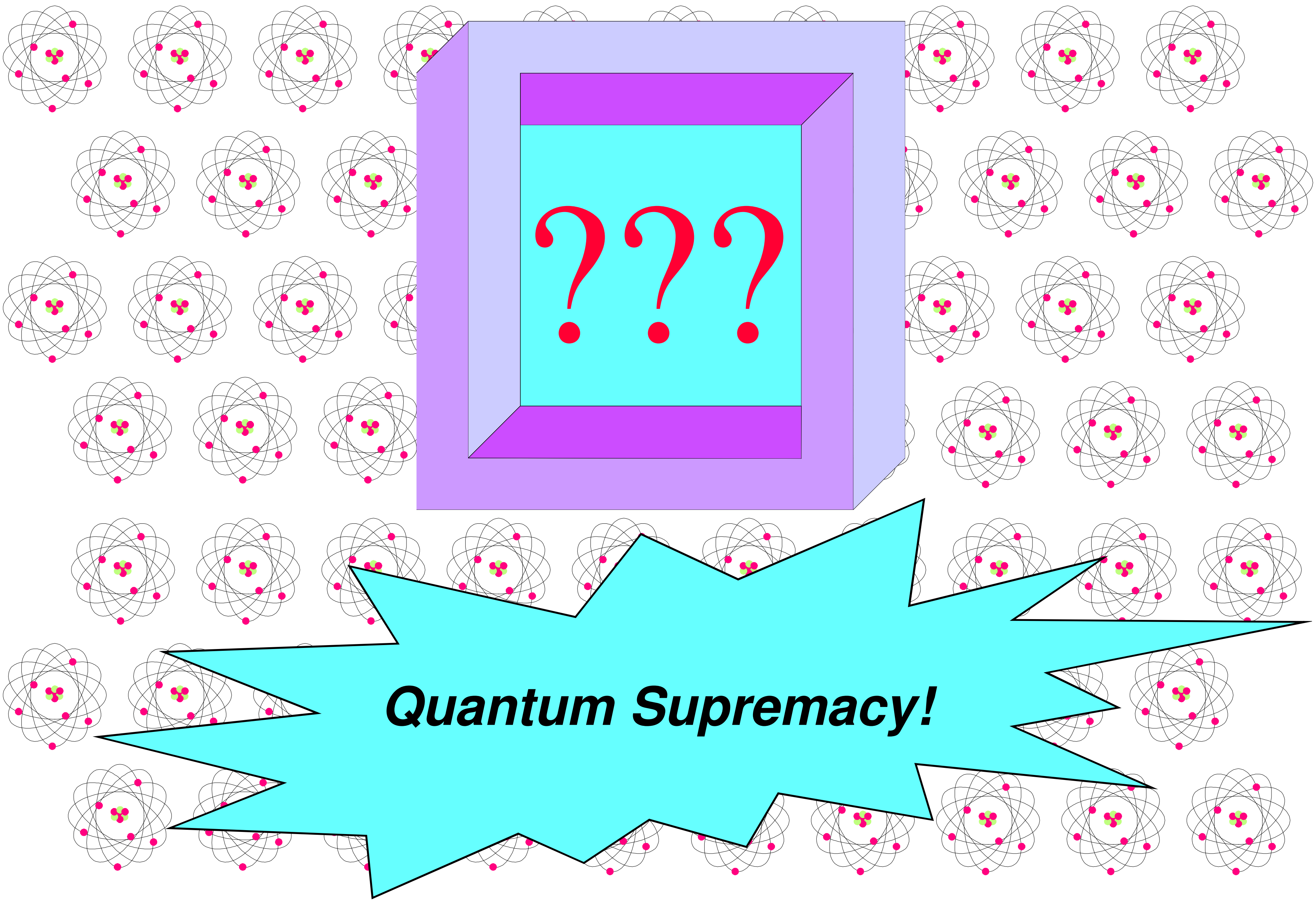
John Preskill

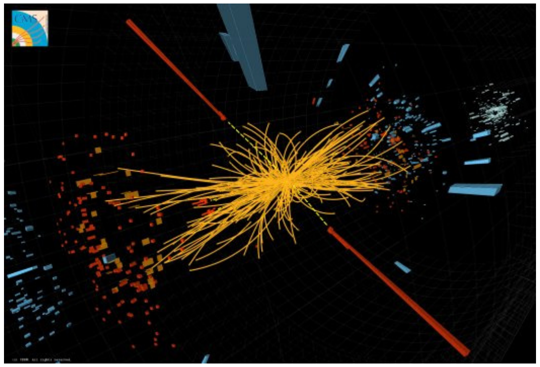
APS March Meeting

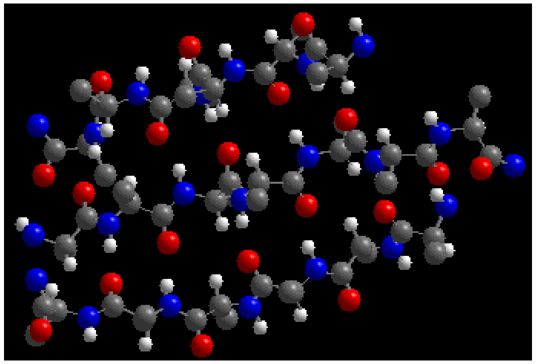
16 March 2016

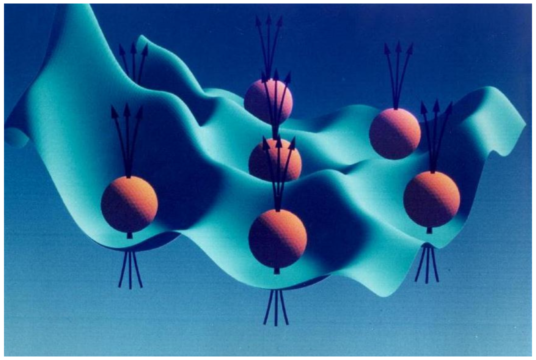
Frontiers of Physics

short distance	long distance	complexity
		
Higgs boson	Large scale structure	“More is different”
Neutrino masses	Cosmic microwave background	Many-body entanglement
Supersymmetry	Dark matter	Phases of quantum matter
Quantum gravity	Dark energy	Quantum computing
String theory	Gravitational waves	Quantum spacetime


???
Quantum Supremacy!

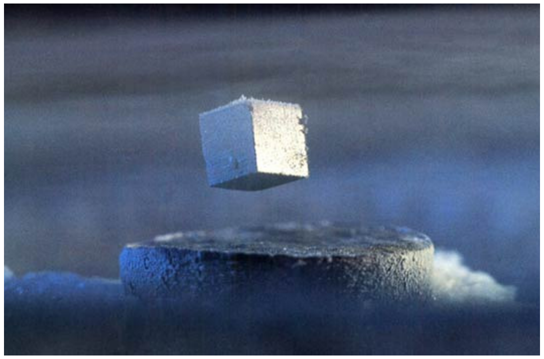

particle collision

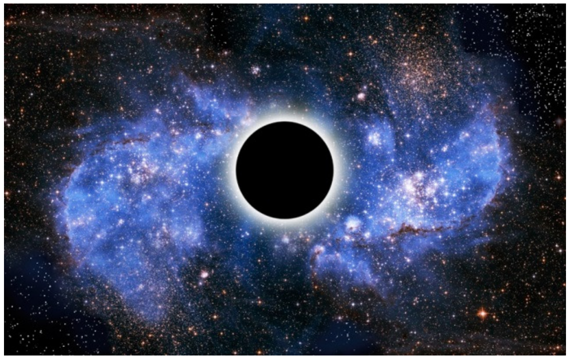

molecular chemistry


entangled electrons

A quantum computer can simulate efficiently any physical process that occurs in Nature.

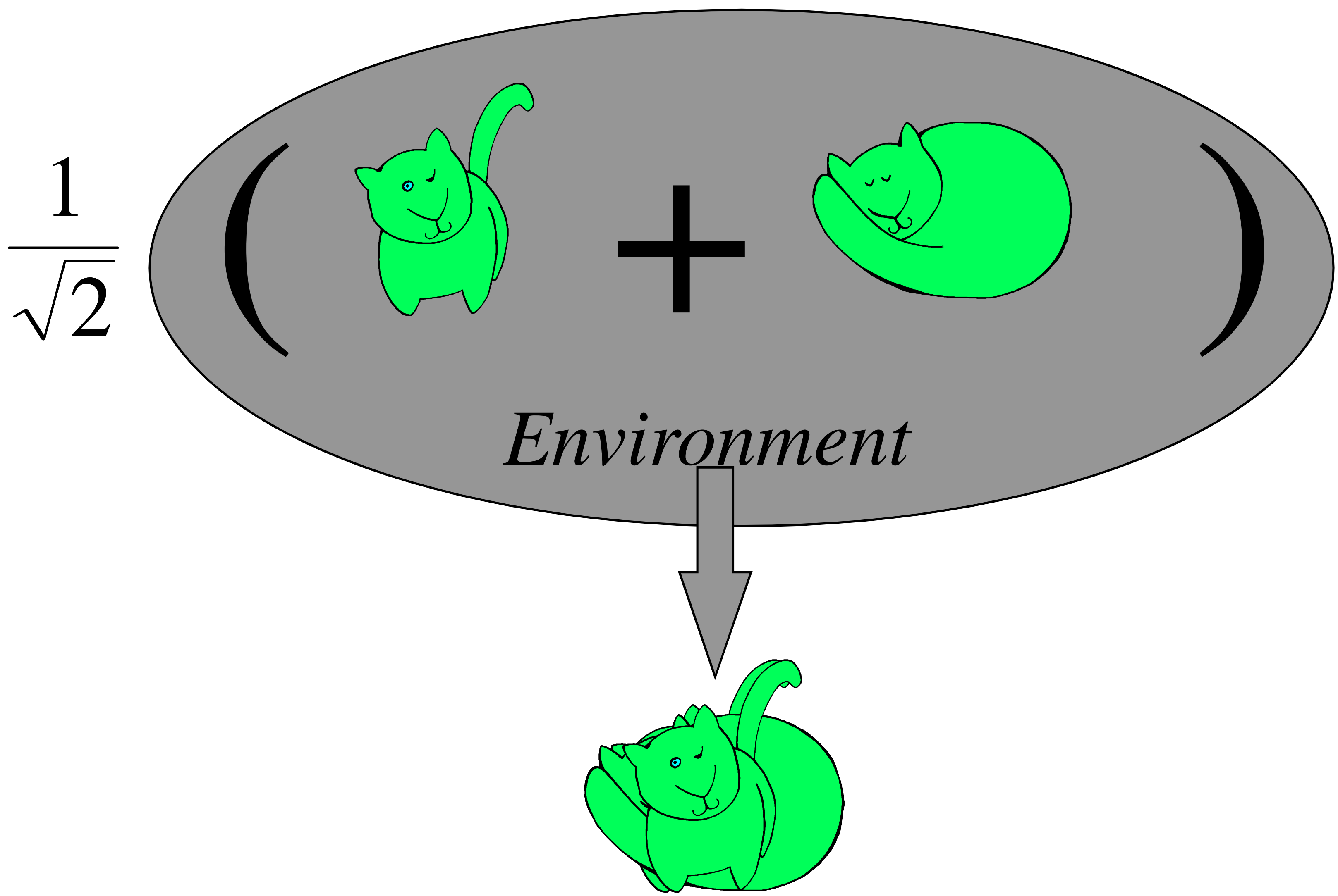
(Maybe. We don't actually know for sure.)


superconductor

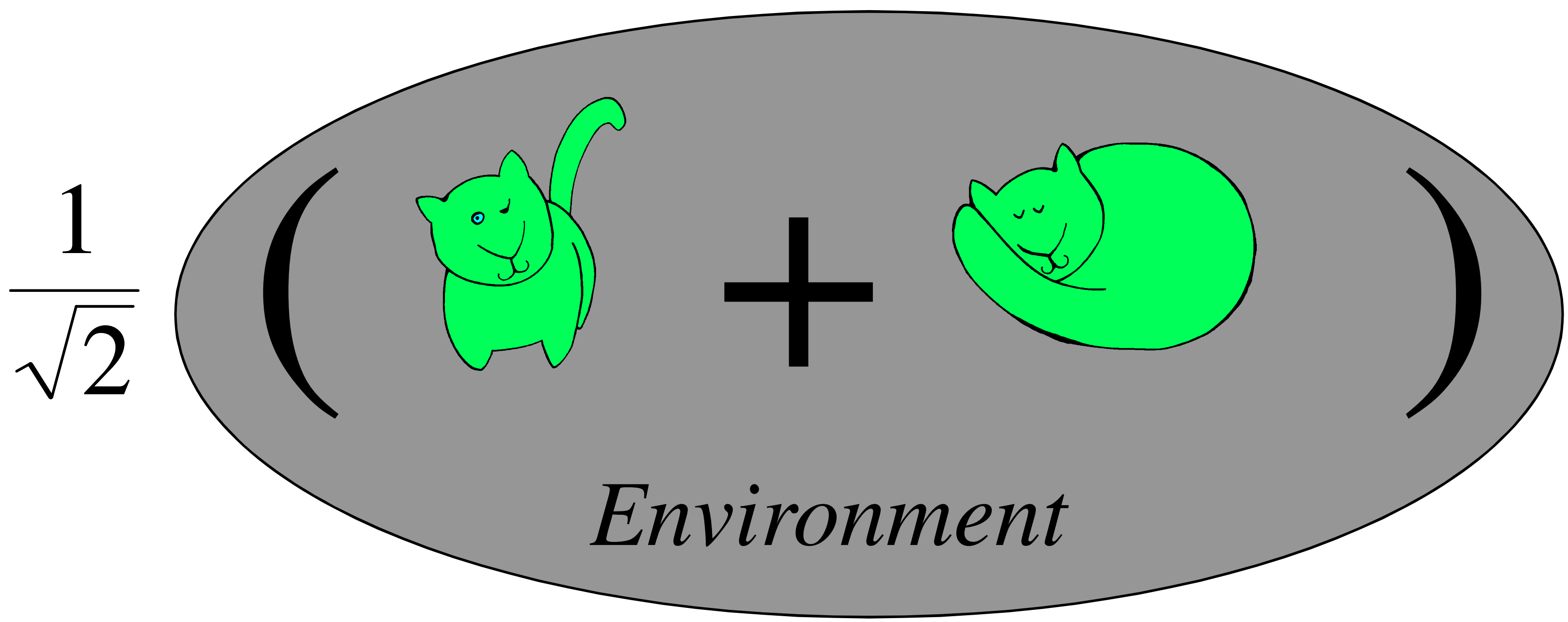

black hole


early universe

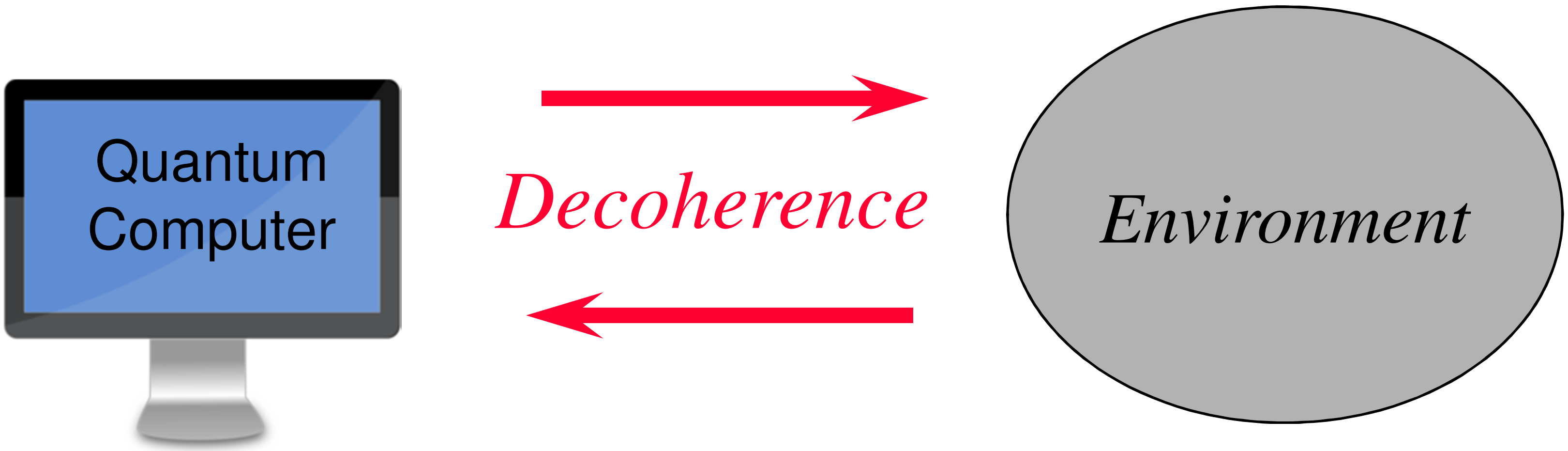
Decoherence



Decoherence



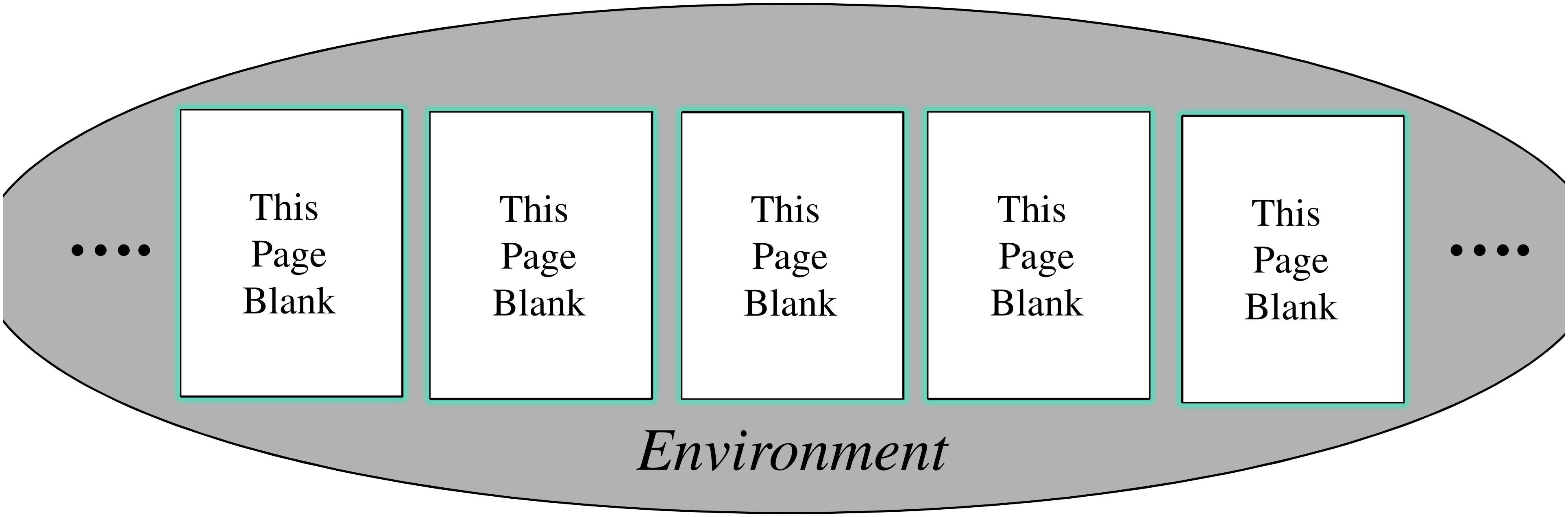
Decoherence explains why quantum phenomena, though observable in the microscopic systems studied in the physics lab, are not manifest in the macroscopic physical systems that we encounter in our ordinary experience.



ERROR!

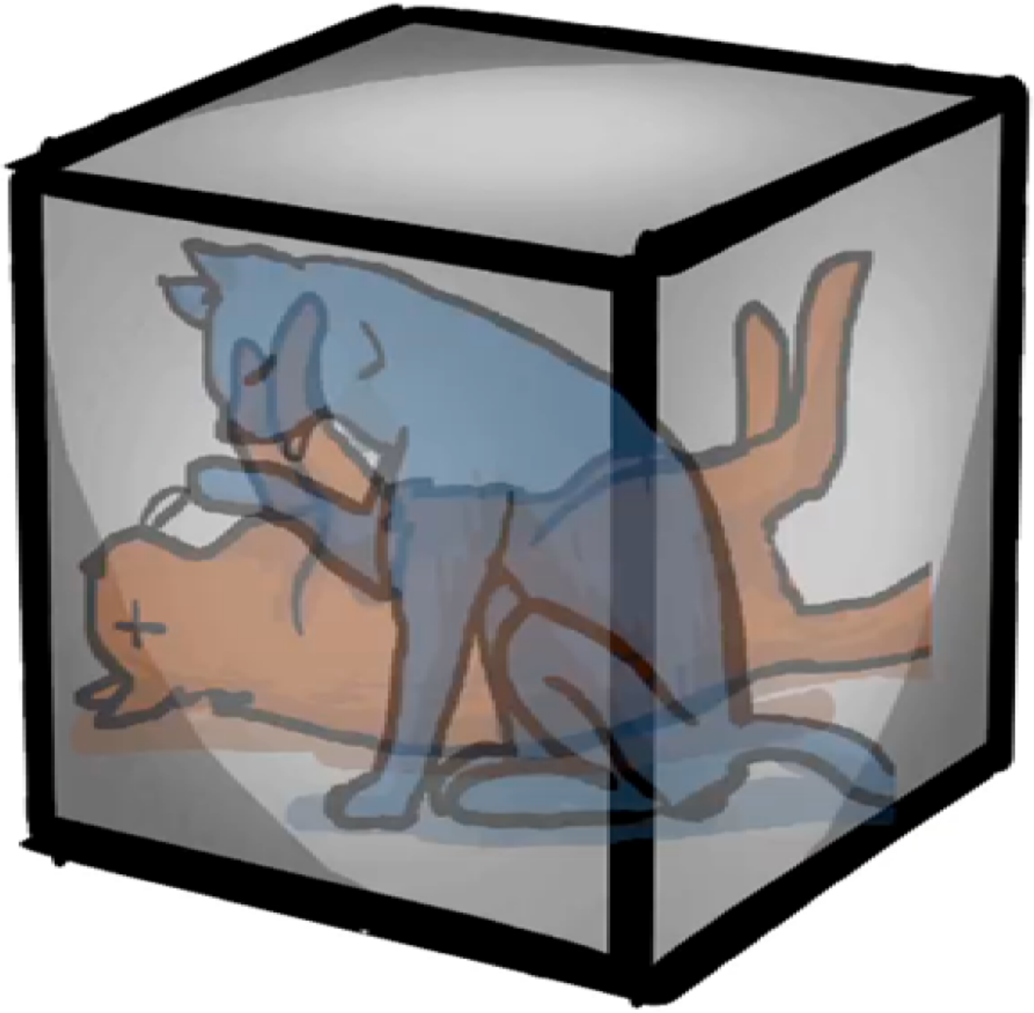
To resist decoherence, we must prevent the environment from "learning" about the state of the quantum computer during the computation.

Quantum error correction



The protected "logical" quantum information is encoded in a highly entangled state of many physical qubits.

The environment can't access this information if it interacts locally with the protected system.



Unruh, Physical Review A,
Submitted June 1994

PHYSICAL REVIEW A VOLUME 51, NUMBER 2 FEBRUARY 1995

Maintaining coherence in quantum computers

W. G. Unruh*
Canadian Institute for Advanced Research, Cosmology Program, Department of Physics,
University of British Columbia, Vancouver, Canada V6T 1Z1
(Received 10 June 1994)

The effects of the inevitable coupling to external degrees of freedom of a quantum computer are examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_B T$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

PACS number(s): 03.65.−w

"The thermal time scale thus sets a (weak) limit on the length of time that a quantum calculation can take."

Landauer, Philosophical Transactions, Published December 1995

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MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

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Is Quantum Mechanics Useful?

Rolf Landauer

Published 15 December 1995. DOI: 10.1098/rsta.1995.0106

"...small errors will accumulate and cause the computation to go off track."

Haroche and Raimond, Physics Today, Published August 1996

QUANTUM COMPUTING: DREAM OR NIGHTMARE?

Recent experiments have deepened our insight into the wonderfully counterintuitive quantum theory. But are they really harbingers of quantum computing? We doubt it.

Serge Haroche and Jean-Michel Raimond

The principles of quantum computing were laid out about 15 years ago by computer scientists applying the superposition principle of quantum mechanics to computer operation. Quantum computing has recently become a hot topic in physics, with the recognition that a two-level system can be pre-

two interacting qubits: a "control" bit and a "target" bit. The control remains unchanged, but its state determines the evolution of the target: If the control is 0, nothing happens to the target; if it is 1, the target undergoes a well-defined transformation.

Quantum mechanics admits additional options. If

Therefore we think it fair to say that, unless some unforeseen new physics is discovered, the implementation of error-correcting codes will become exceedingly difficult as soon as one has to deal with more than a few gates. In this sense the large-scale quantum machine, though it may be the computer scientist's dream, is the experimenter's nightmare.

Peter
Shor

PHYSICAL REVIEW A
ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

THIRD SERIES, VOLUME 52, NUMBER 4 OCTOBER 1995

RAPID COMMUNICATIONS

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication should be no longer than 4 printed pages and must be accompanied by an abstract. Page proofs are sent to authors.

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*
AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 17 May 1995)

Combining repetition codes for bit flips and phase errors (Shor code).

PHYSICAL REVIEW LETTERS

VOLUME 77 29 JULY 1996 NUMBER 5

Error Correcting Codes in Quantum Theory

A. M. Steane
Clarendon Laboratory, Parks Road, Oxford, OX1 3PU, England
(Received 4 October 1995)

A quantum version of the classical Hamming code (Steane code).

VOLUME 76, NUMBER 5 PHYSICAL REVIEW LETTERS 29 JANUARY 1996

Purification of Noisy Entanglement and Faithful Teleportation via Noisy Channels

Charles H. Bennett,[1,*] Gilles Brassard,[2,†] Sandu Popescu,[3,‡] Benjamin Schumacher,[4,§]
John A. Smolin,[5,‖] and William K. Wootters[6,¶]

[1]*IBM Research Division, Yorktown Heights, New York 10598*
[2]*Département IRO, Université de Montréal, C.P. 6128, Succursale centre-ville, Montréal, Québec, Canada H3C 3J7*
[3]*Physics Department, Tel Aviv University, Tel Aviv, Israel*
[4]*Physics Department, Kenyon College, Gambier, Ohio 43022*
[5]*Physics Department, University of California at Los Angeles, Los Angeles, California 90024*
[6]*Physics Department, Williams College, Williamstown, Massachusetts 01267*

$\boxed{\text{(Received 24 April 1995)}}$

Two separated observers, by applying local operations to a supply of not-too-impure entangled states (e.g., singlets shared through a noisy channel), can prepare a smaller number of entangled pairs of arbitrarily high purity (e.g., near-perfect singlets). These can then be used to faithfully teleport unknown quantum states from one observer to the other, thereby achieving faithful transmission of quantum information through a noisy channel. We give upper and lower bounds on the yield $D(M)$ of pure singlets ($|\Psi^-\rangle$) distillable from mixed states M, showing $D(M) > 0$ if $\langle\Psi^-|M|\Psi^-\rangle > \frac{1}{2}$.

PACS numbers: 03.65.Bz, 42.50.Dv, 89.70.+c

Entanglement purification and teleportation for faithful transmission of quantum information through noisy channels.

PHYSICAL REVIEW A VOLUME 54, NUMBER 2 AUGUST 1996

Good quantum error-correcting codes exist

A. R. Calderbank and Peter W. Shor
AT&T Research, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 12 September 1995)

Multiple-particle interference and quantum error correction

By Andrew Steane
Department of Atomic and Laser Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK
a.steane@physics.oxford.ac.uk

Proceedings of the Royal Society A, Received 27 November 1995, Published 8 November 1996

Calderbank-Shor-Steane (CSS) Codes: the first family of good quantum codes.

Author: Steane
Title: Multiple particle interference and quantum error correction
Manuscript Number: 95PA342

This paper is a major contribution to quantum information theory, one of the most significant in recent years. It contains deep and surprising new results, and it is clearly written. Without question, it is worthy of publication in the Proceedings.

PHYSICAL REVIEW A VOLUME 54, NUMBER 3 SEPTEMBER 1996

Class of quantum error-correcting codes saturating the quantum Hamming bound

Daniel Gottesman*
California Institute of Technology, Pasadena, California 91125
(Received 29 April 1996)

I develop methods for analyzing quantum error-correcting codes, and use these methods to construct an infinite class of codes saturating the quantum Hamming bound. These codes encode $k=n-j-2$ quantum bits (qubits) in $n=2^j$ qubits and correct $t=1$ error. [S1050-2947(96)09309-2]

PACS number(s): 03.65.Bz, 89.80.+h

VOLUME 78, NUMBER 3 PHYSICAL REVIEW LETTERS 20 JANUARY 1997

Quantum Error Correction and Orthogonal Geometry

A. R. Calderbank,[1] E. M. Rains,[2] P. W. Shor,[1] and N. J. A. Sloane[1]
[1]*AT&T Labs—Research, Murray Hill, New Jersey 07974*
[2]*Institute for Defense Analyses, Princeton, New Jersey 08540*
(Received 9 May 1996; revised manuscript received 3 July 1996)

A group theoretic framework is introduced that simplifies the description of known quantum error-correcting codes and greatly facilitates the construction of new examples. Codes are given which map 3 qubits to 8 qubits correcting 1 error, 4 to 10 qubits correcting 1 error, 1 to 13 qubits correcting 2 errors, and 1 to 29 qubits correcting 5 errors. [S0031-9007(96)02177-1]

Quantum stabilizer codes:

the quantum analogue of additive classical codes.

Search or

Quantum Physics

Fault-tolerant quantum computation

Peter W. Shor (AT&T Research)

(Submitted on 13 May 1996 (v1), last revised 5 Mar 1997 (this version, v2))

Recently, it was realized that use of the properties of quantum mechanics might speed up certain computations dramatically. Interest in quantum computation has since been growing. One of the main difficulties of realizing quantum computation is that decoherence tends to destroy the information in a superposition of states in a quantum computer, thus making long computations impossible. A futher difficulty is that inaccuracies in quantum state transformations throughout the computation accumulate, rendering the output of long computations unreliable. It was previously known that a quantum circuit with t gates could tolerate O(1/t) amounts of inaccuracy and decoherence per gate. We show, for any quantum computation with t gates, how to build a polynomial size quantum circuit that can tolerate O(1/(log t)^c) amounts of inaccuracy and decoherence per gate, for some constant c. We do this by showing how to compute using quantum error correcting codes. These codes were previously known to provide resistance to errors while storing and transmitting quantum data.

Comments: Latex, 11 pages, no figures, in 37th Symposium on Foundations of Computing, IEEE Computer Society Press, 1996, pp. 56-65

Fault-tolerant syndrome measurement, using encoded ancillas, verified offline.

Universal gates acting on encoded quantum data, using "magic states" verified offline.

arXiv.org > quant-ph > arXiv:quant-ph/9610011

Search or

Quantum Physics

Threshold Accuracy for Quantum Computation

E. Knill, R. Laflamme, W. Zurek

(Submitted on 8 Oct 1996 (v1), last revised 15 Oct 1996 (this version, v3))

We have previously (quant-ph/9608012) shown that for quantum memories and quantum communication, a state can be transmitted over arbitrary distances with error ϵ provided each gate has error at most $c\epsilon$. We discuss a similar concatenation technique which can be used with fault tolerant networks to achieve any desired accuracy when computing with classical initial states, provided a minimum gate accuracy can be achieved. The technique works under realistic assumptions on operational errors. These assumptions are more general than the stochastic error heuristic used in other work. Methods are proposed to account for leakage errors, a problem not previously recognized.

arXiv.org > quant-ph > arXiv:quant-ph/9611025

Search or A

Quantum Physics

Fault Tolerant Quantum Computation with Constant Error

Dorit Aharonov (Physics and computer science, Hebrew Univ.), Michael Ben-Or (Computer science, Hebrew univ.)

(Submitted on 14 Nov 1996 (v1), last revised 15 Nov 1996 (this version, v2))

Recently Shor showed how to perform fault tolerant quantum computation when the error probability is logarithmically small. We improve this bound and describe fault tolerant quantum computation when the error probability is smaller than some constant threshold. The cost is polylogarithmic in time and space, and no measurements are used during the quantum computation. The result holds also for quantum circuits which operate on nearest neighbors only. To achieve this noise resistance, we use concatenated quantum error correcting codes. The scheme presented is general, and works with all quantum codes that satisfy some restrictions, namely that the code is ``proper''.

Scalable quantum computing using recursive simulations.

Scalable quantum computing

Quantum Accuracy Threshold Theorem: Consider a quantum computer subject to quasi-independent noise with strength ε. There exists a constant $\varepsilon_0 > 0$ such that for a fixed $\varepsilon < \varepsilon_0$ and fixed $\delta > 0$, any circuit of size L can be simulated by a circuit of size L^* with accuracy greater than $1-\delta$, where, for some constant c,

$$L^* = O\left[L(\log L)^c \right]$$

Aharonov, Ben-Or
Kitaev
Laflamme, Knill, Zurek
Aliferis, Gottesman, Preskill
Reichardt

assuming:

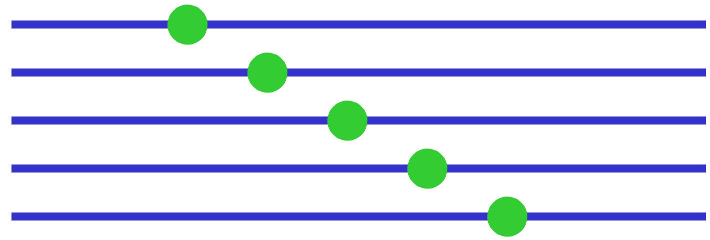
parallelism, fresh qubits (*necessary* assumptions)

nonlocal gates, fast measurements, fast and accurate classical processing, no leakage (*convenient* assumptions).

“Practical” considerations:
Resource requirements, systems engineering issues

Matters of “principle”:
Conditions on the noise model, what schemes are scalable, etc.

Limitations on transversal (local unitary) logical gates



The logical gates close to the identity that can be executed with local unitary transformations form a (perhaps trivial) Lie algebra.

$$U = I + \varepsilon(A_1 + A_2 + \cdots + A_n)$$

If the code can "detect" a weight-one error, then for each i:

$$\Pi A_i \Pi \propto \Pi$$

(Π= projector onto code space)

If U preserves the code space, then U acts trivially on code space:

$$U\,\Pi = \Pi\, U\, \Pi = \Pi$$

There are no transversal gates close to the identity. The group generated by transversal gates is finite and hence nonuniversal. Which logical gates can be executed transversally depends on what code we use.

Eastin-Knill 2009

Evading the Eastin-Knill Theorem

Measurements, i.e. ancillas as "quantum software". Shor (1996), Gottesman& Chuang (1999), Bravyi and Kitaev (2005).

Multiple partitions. Jochym-O'Connor and Laflamme (2013). One set of operations transversal with respect to one partition and another set with respect to another.

$$\langle \mathcal{L}_1, \mathcal{L}_2 \rangle = \text{all}$$

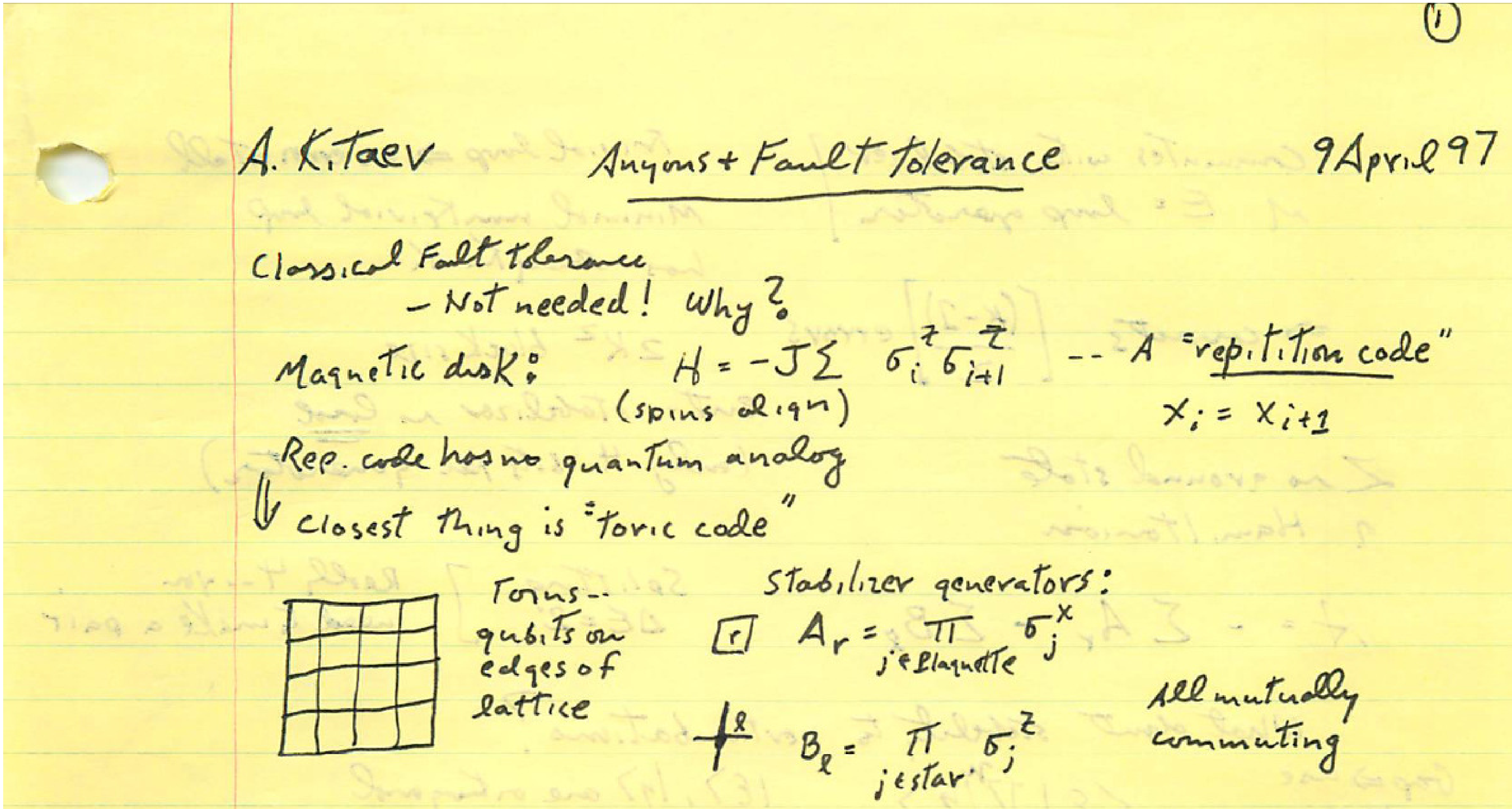
Code switching. Duclos-Cianci and Poulin (2014). For example, fix the gauge of a subsystem code in two distinct ways. Need to switch fault-tolerantly.

$$\mathcal{C}_1 \leftrightarrow \mathcal{C}_2$$

Code drift. Paetznick and Reichardt (2013). Need to return to the original code fault-tolerantly.

$$\mathcal{L} : \mathcal{C}_1 \to \mathcal{C}_2$$

Alexei
Kitaev



A. Kitaev — Anyons + Fault Tolerance — 9 April 97 — 1

Classical Fault Tolerance

– Not needed! Why?

Magnetic disk: $H = -J\sum \sigma_i^z \sigma_{i+1}^z$ -- A "repitition code"

(spins align) $x_i = x_{i+1}$

Rep. code has no quantum analog

⇓ closest thing is "toric code"

Torus -- qubits on edges of lattice

Stabilizer generators:

$\boxed{r}$ $A_r = \prod_{j \in \text{plaquette}} \sigma_j^x$

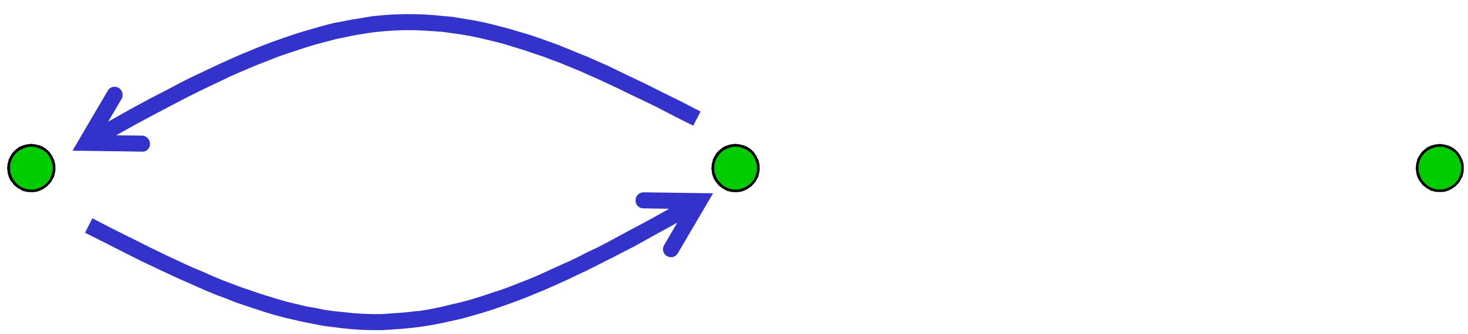
$B_\ell = \prod_{j \in \text{star}} \sigma_j^z$

All mutually commuting

9 April 1997 … An exciting day!

Nonabelian anyons

Quantum information can be stored in the collective state of exotic particles in two spatial dimensions (“anyons”).



The information can be processed by exchanging the positions of the anyons (even though the anyons never come close to one another).

Nonabelian anyons

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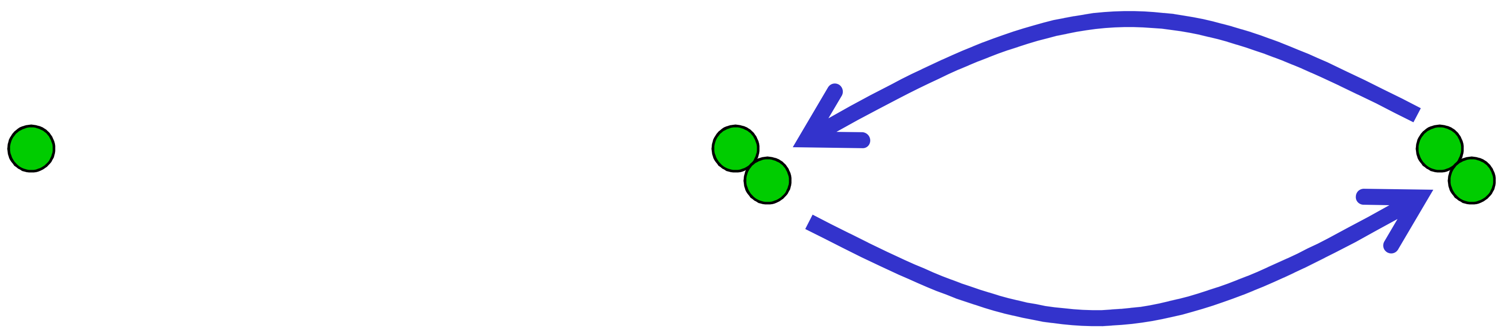
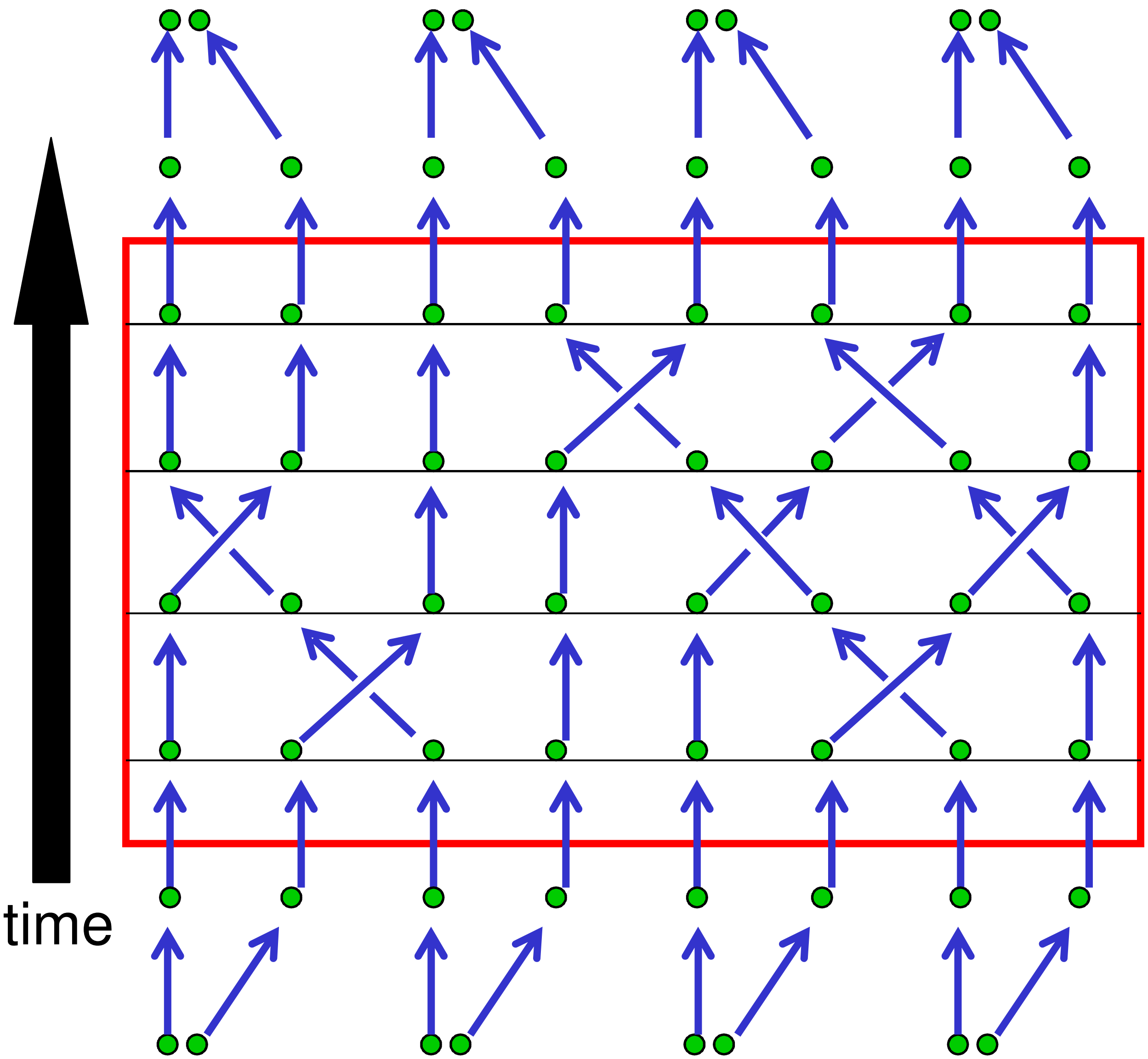


The information can be processed by exchanging the positions of the anyons (even though the anyons never come close to one another).

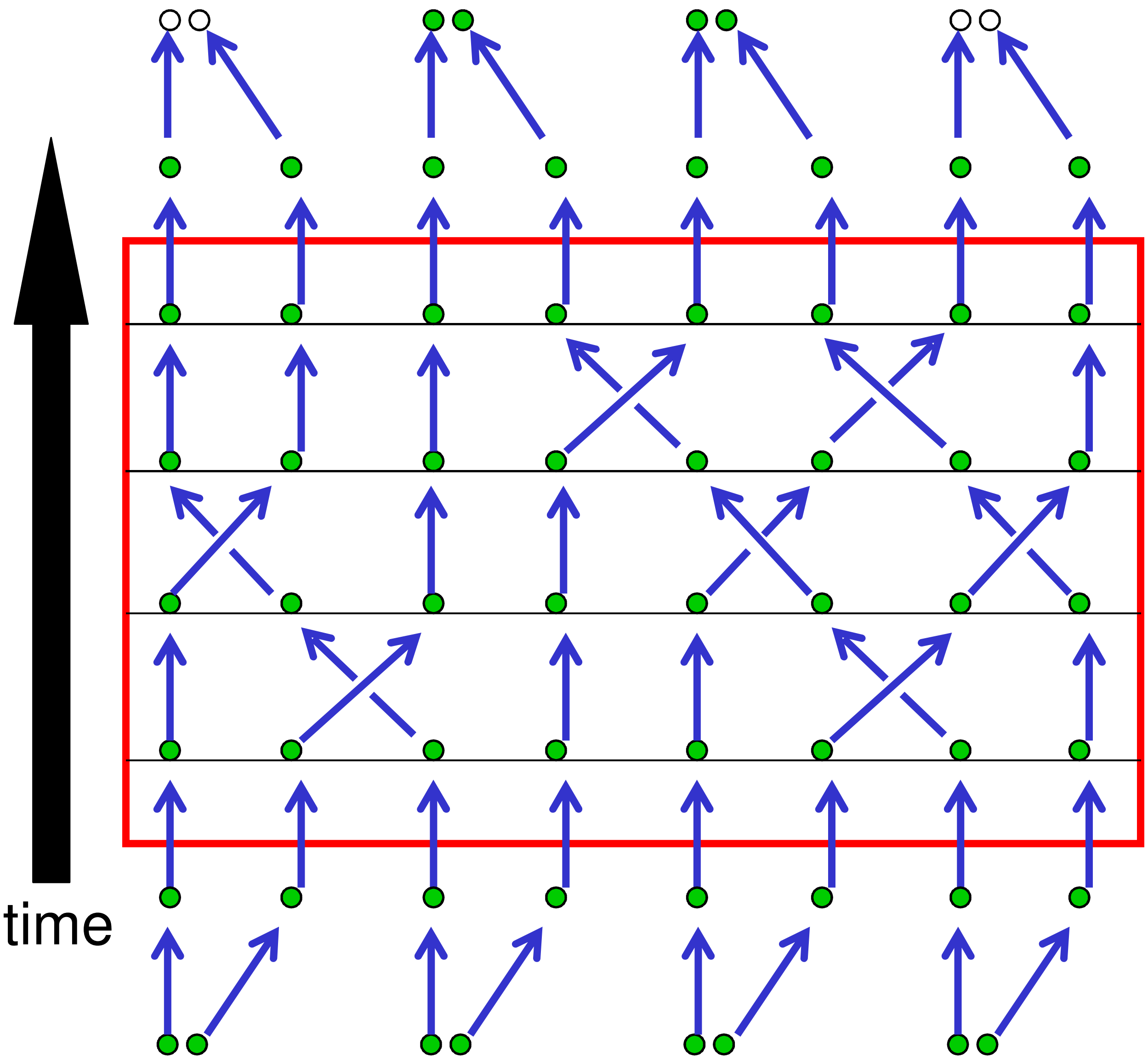
Topological quantum computation (Kitaev '97, FLW '00)



Kitaev

Freedman

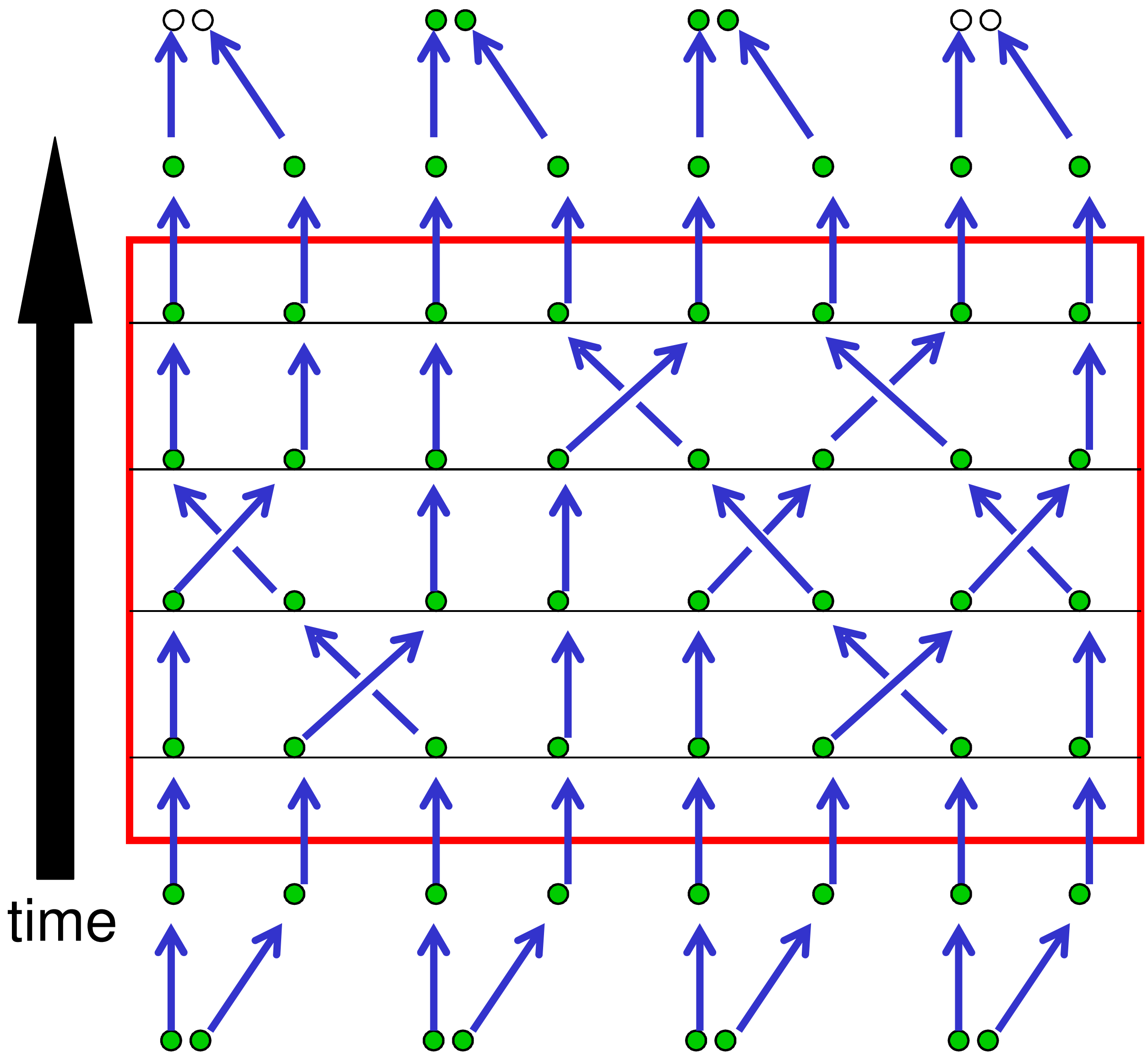
Topological quantum computation (Kitaev '97, FLW '00)



Kitaev

Freedman

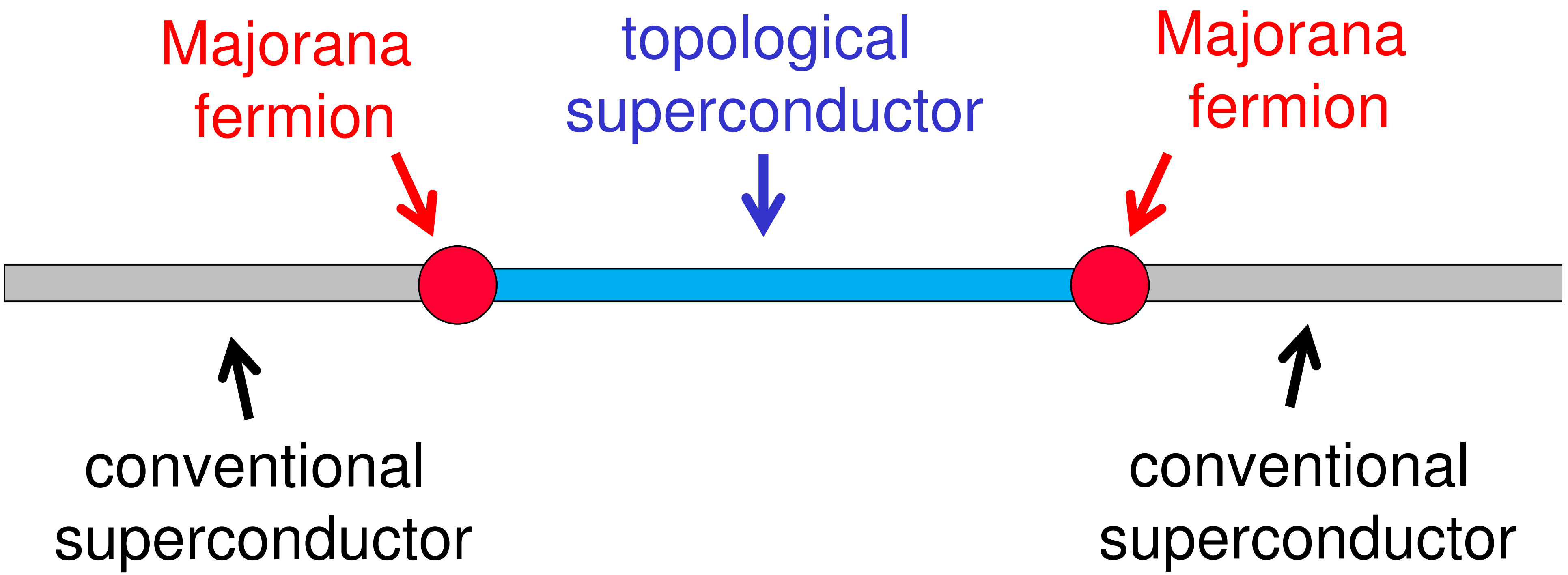
Topological quantum computation

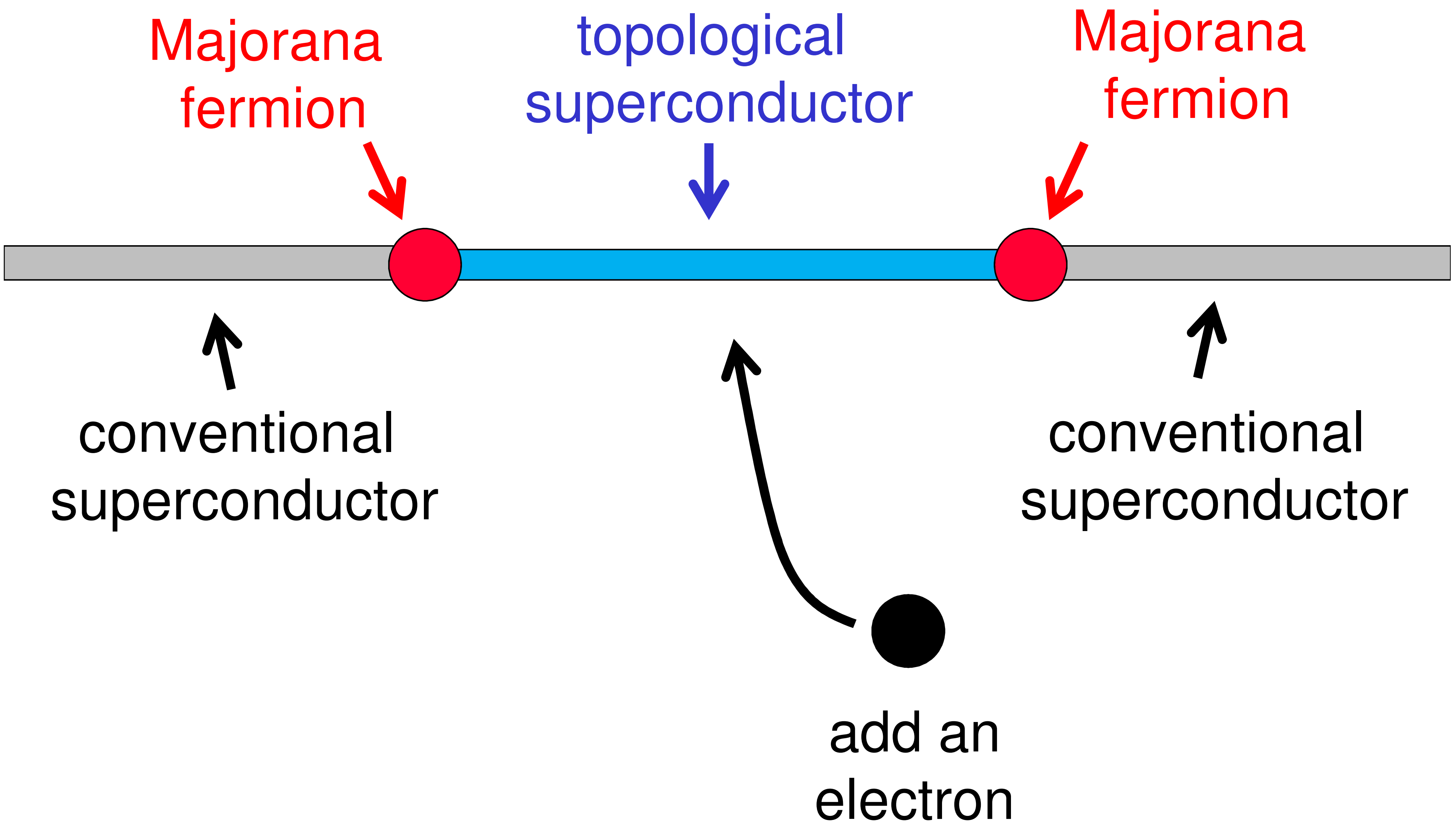


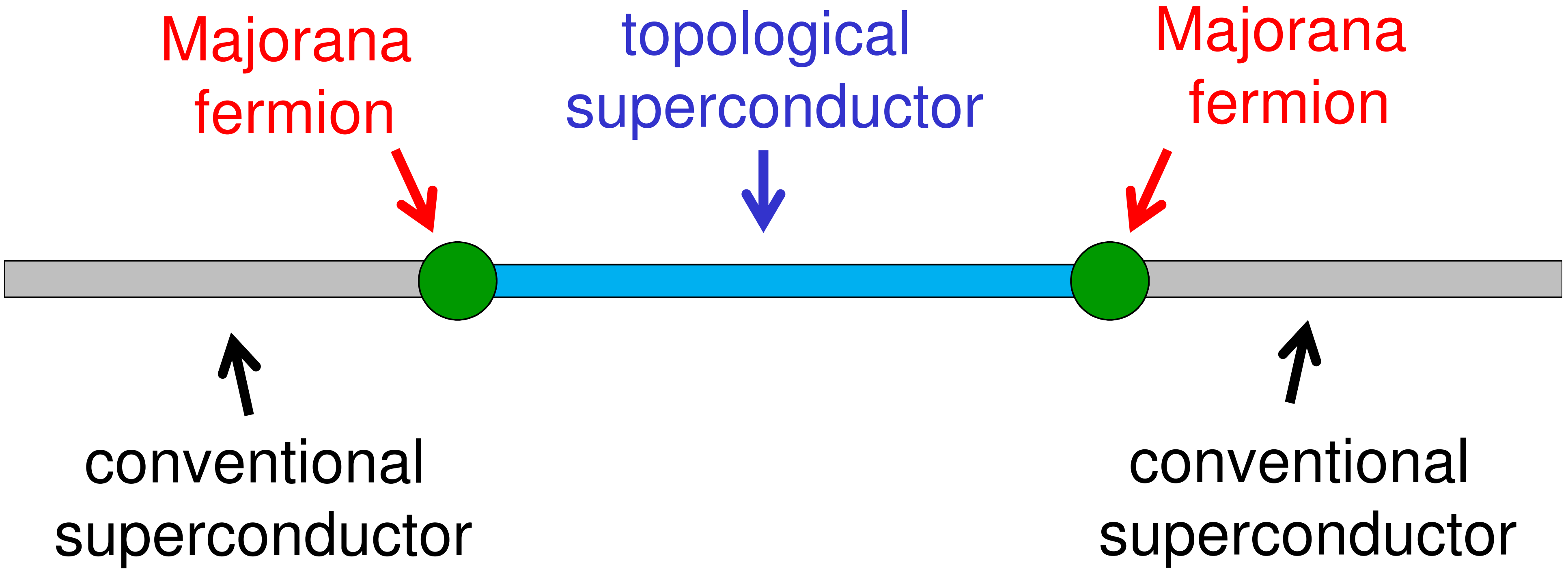
The computation is intrinsically resistant to decoherence.

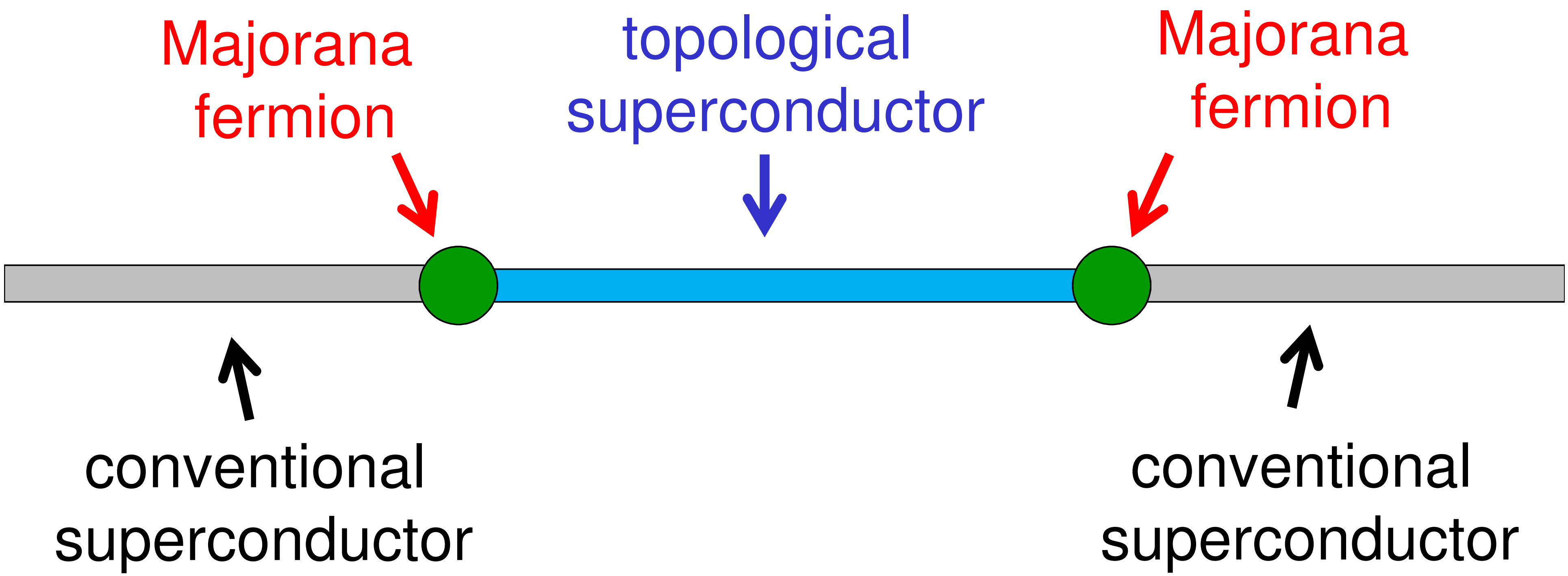
If the paths followed by the particles in spacetime execute the right braid, then the quantum computation is guaranteed to give the right answer!

Kitaev's (2001) magic trick: sawing an *electron* in half!

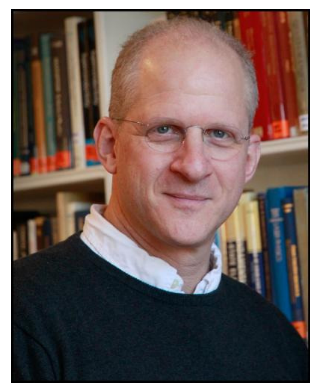

Majorana
fermion
topological
superconductor
Majorana
fermion
conventional
superconductor
conventional
superconductor


Majorana fermion
topological superconductor
Majorana fermion
conventional superconductor
conventional superconductor
add an electron


Majorana
fermion
topological
superconductor
Majorana
fermion
conventional
superconductor
conventional
superconductor



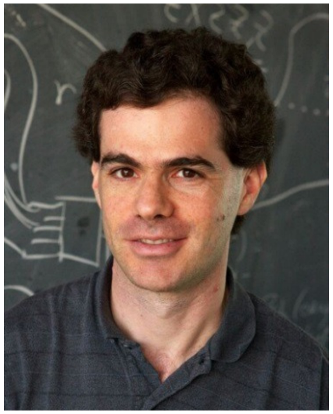
Kouwenhoven



Marcus

Mourik, Zuo, Frolov, Plissard, Bakkers, Kouwenhoven (2012).
Albrecht, Higginbotham, Madsen, Kuemmeth, Jespersen, Nygard, Krogstrup, Marcus (2016).

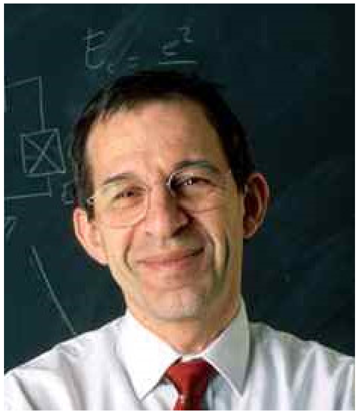
Daniel Gottesman,
Fault tolerance in small experiments



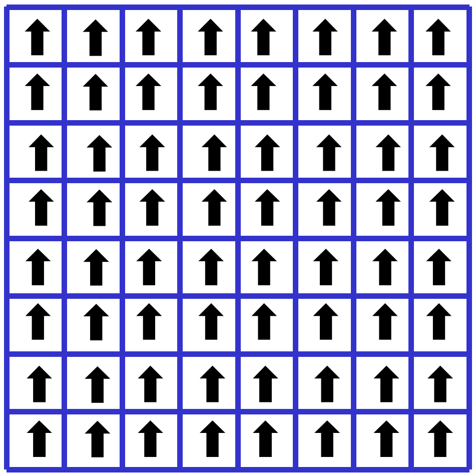
David DiVincenzo,
Quantum error correction and the future of solid state quantum computing

Philipp Schindler,
Quantum error correction with trapped ions

Michel Devoret,
Quantum error correction in superconducting circuits

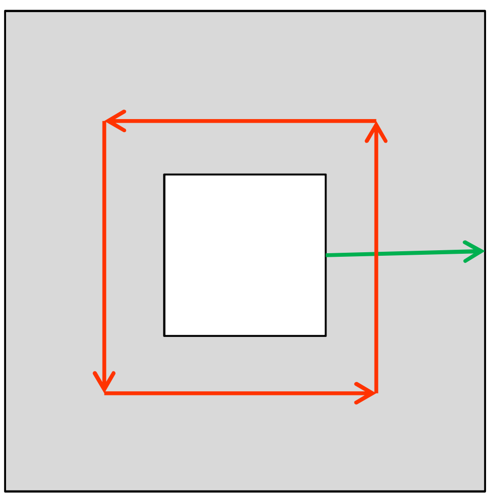


Topological quantum error correction



Classical memory ≈ ferromagnet order

Robust bit



Quantum memory ≈ topological order

Robust qubit

Red or green (abelian) anyons

Realize physically, or simulate with generic hardware.

Surface code: Dennis, Landahl, Kitaev, Preskill (2002), Raussendorf, Harrington, and Goyal (2007).

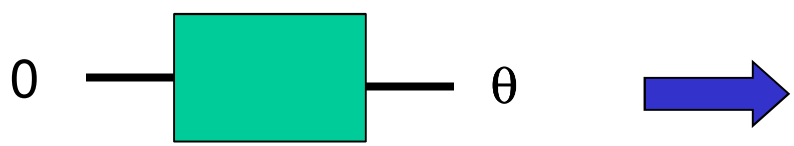
Protected Hardware

The best way to reduce the overhead cost of fault-tolerance: better gates!

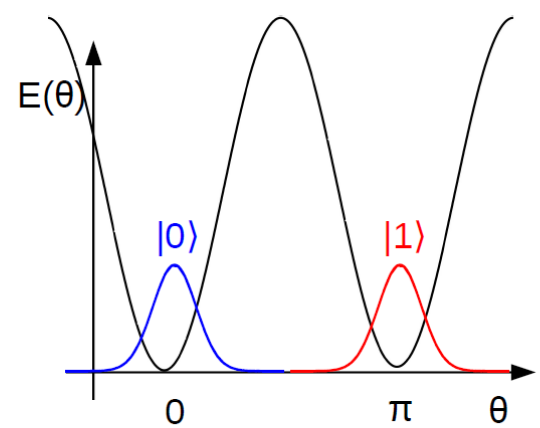
Gradually the distinction between error correction in “hardware” and “software” will blur. We will learn to make better gates in many platforms by incorporating error correction at the physical level.

For example:

“0-Pi qubit” – robust degeneracy in a superconducting circuit:



$$\boxed{E \approx f(2\theta) + O\left(\exp\left(-c(\text{size})\right)\right)}$$



Feigel’man & Ioffe (2002). Doucot & Vidal (2002).
Kitaev (2006). Brooks, Kitaev, Preskill (2013).
Schuster talk.

Self-correcting quantum memory

1) Finite-dimensional spins.

2) Bounded-strength local interactions.

3) Nontrivial codespace.

4) Perturbative stability.

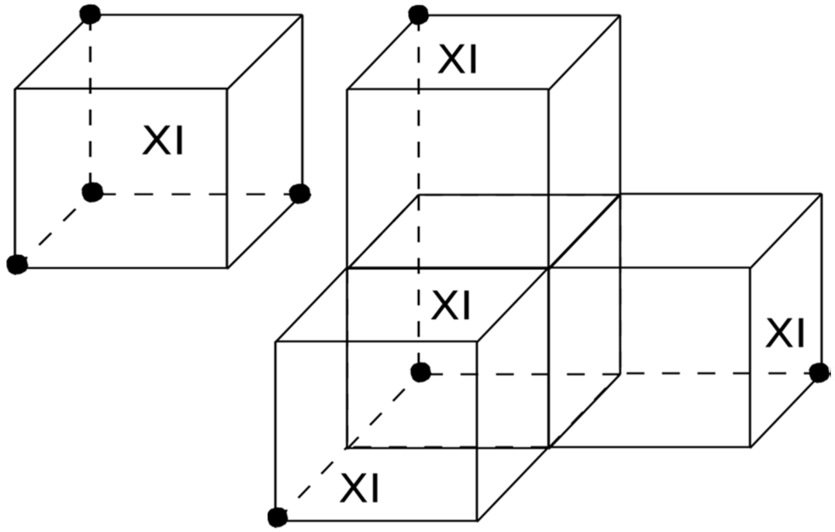
5) Efficient decoding.

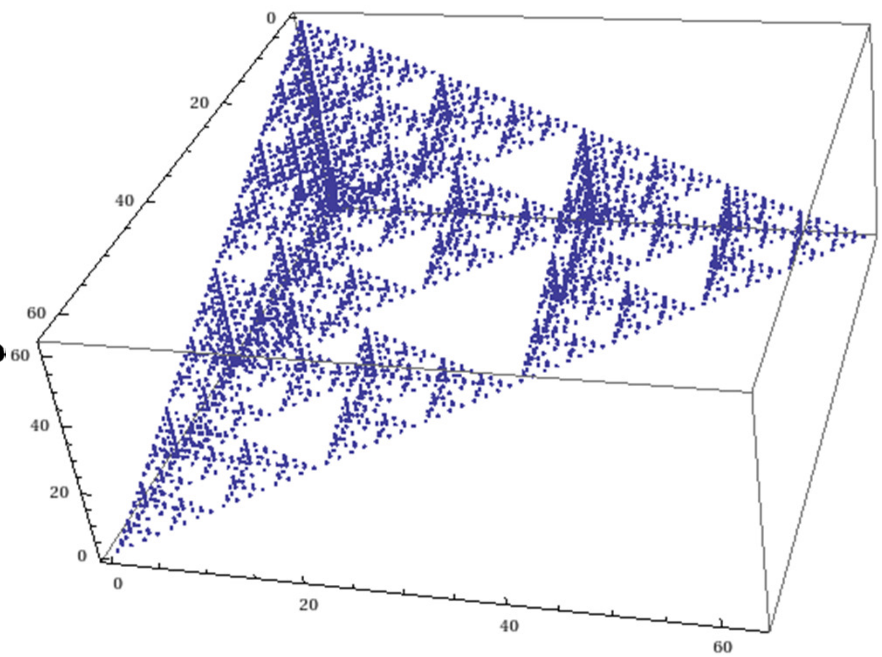
6) Memory time exponential in system size at nonzero temperature.

The 4D toric code obeys all the rules, but what about < 4 dimensions?

Dennis, Landahl, Kitaev, Preskill (2002).

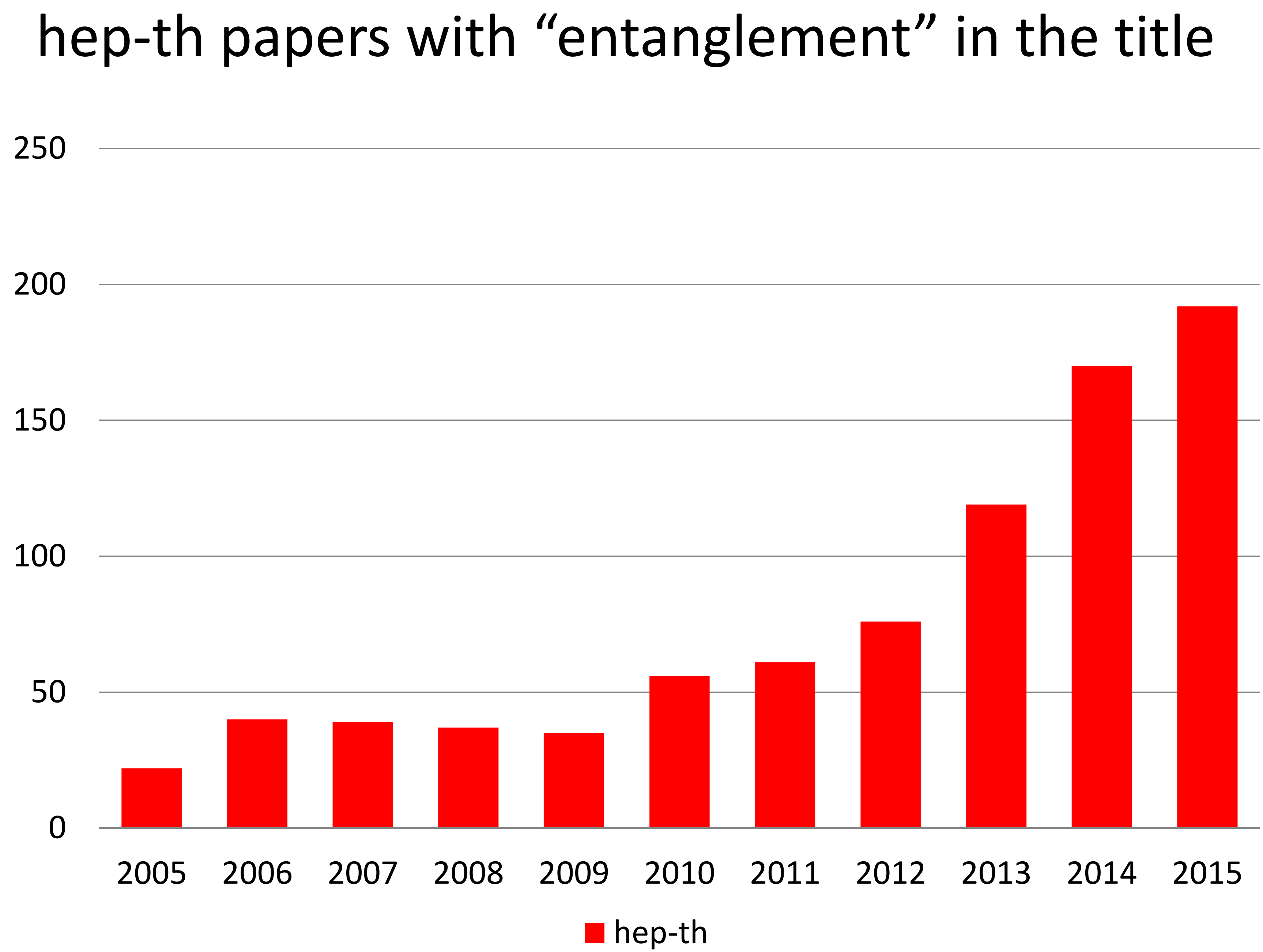
Haah code (2011): Novel 3D topological order



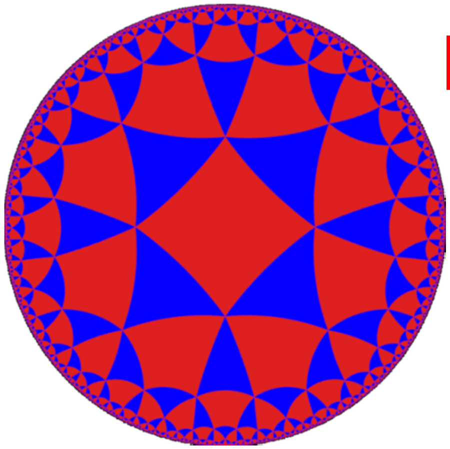


A local process starting from the "vacuum" (no excitations) and arriving at a state where a single topological defect is isolated from all others by distance at least R, must pass through a state whose "energy" is logarithmic in R.

This energy barrier impedes thermal defect diffusion, enhancing the stability of the quantum memory.


hep-th papers with “entanglement” in the title
250
200
150
100
50
0
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
hep-th

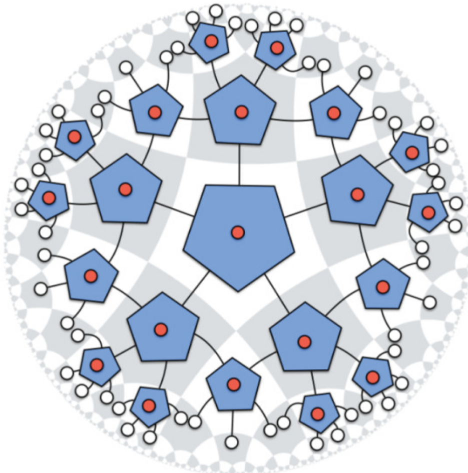
Two amazing ideas:



Holographic correspondence

Quantum error correction

Are they closely related?



-- Scrambled encoding on boundary, protected against erasure.
-- Entanglement seems to be the glue holding space together.
-- Illustrates the surprising unity of physics.
-- Toward accessible experiments probing quantum gravity?

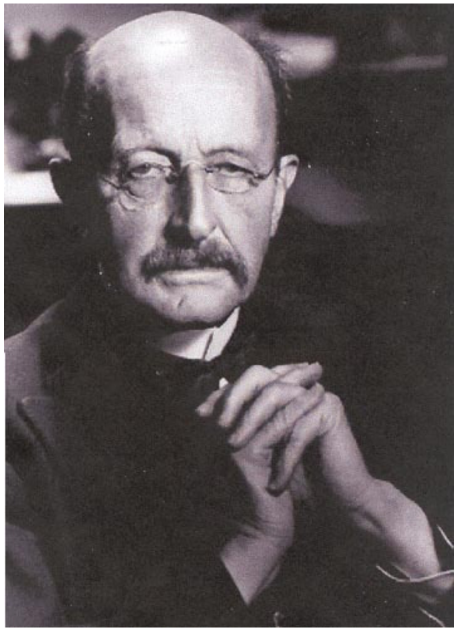
Pastawski

Yoshida

Harlow

Holographic Quantum Error-Correcting Codes, Journal of High Energy Physics 6, 149 (2015).

Building on Almheiri, Dong, Harlow (2014).



PARADOX!

When the theories we use to describe Nature lead to unacceptable or self-contradictory conclusions, we are faced with a great challenges and great opportunities….

Planck
1900

Hawking
1975

“*The ultraviolet catastrophe*”

In thermal equilibrium at nonzero temperature, the electromagnetic field carries an infinite energy per unit volume …

The end of
classical physics!

“*The information loss puzzle*”

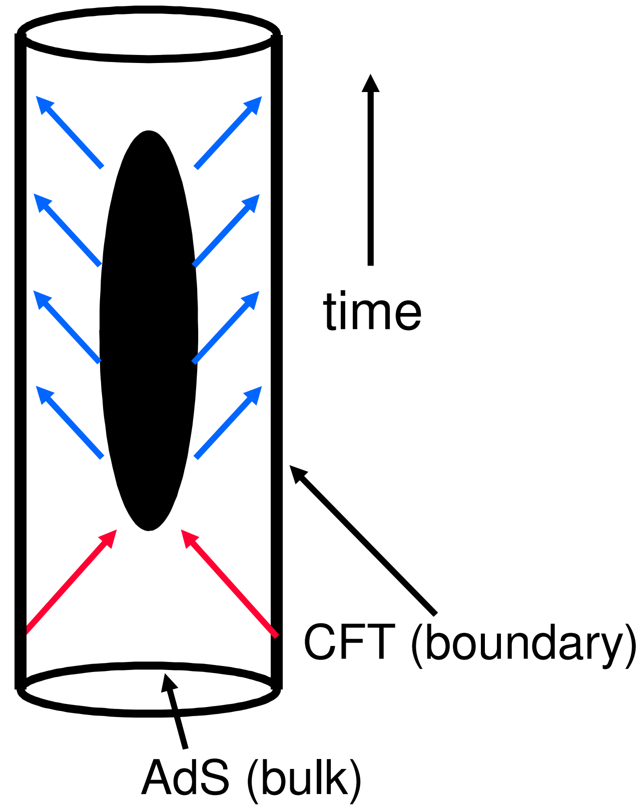
The radiation emitted by an evaporating black hole is featureless, revealing nothing about how the black hole formed …

The end of quantum physics?
(Or of relativistic causality?)

A black hole in a bottle

We can describe the formation and evaporation of a black hole using an "ordinary" quantum theory on the walls of the bottle, where information has nowhere to hide (*Maldacena 1997*).

A concrete realization of the "holographic principle" (*'t Hooft 1994, Susskind 1994*).



So at least in the one case where we think we understand how quantum gravity works, a black hole seems not to destroy information!

Even so, the mechanism by which information can escape from behind a putative event horizon remains murky.

Indeed, it is not clear whether or how the boundary theory describes the experience of observers who cross into the black hole interior, or even if there is an interior!

Bulk/boundary duality: an *exact* correspondence

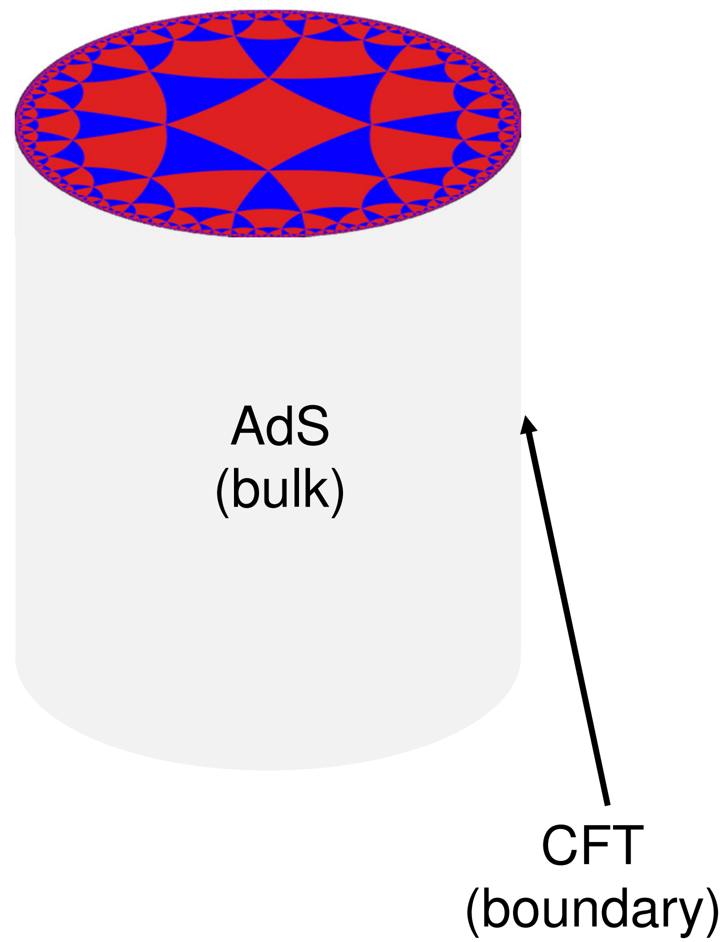
-- Weakly-coupled gravity in the bulk $\leftrightarrow$ strongly-coupled conformal field theory on boundary.

-- Complex dictionary maps bulk operators to boundary operators.

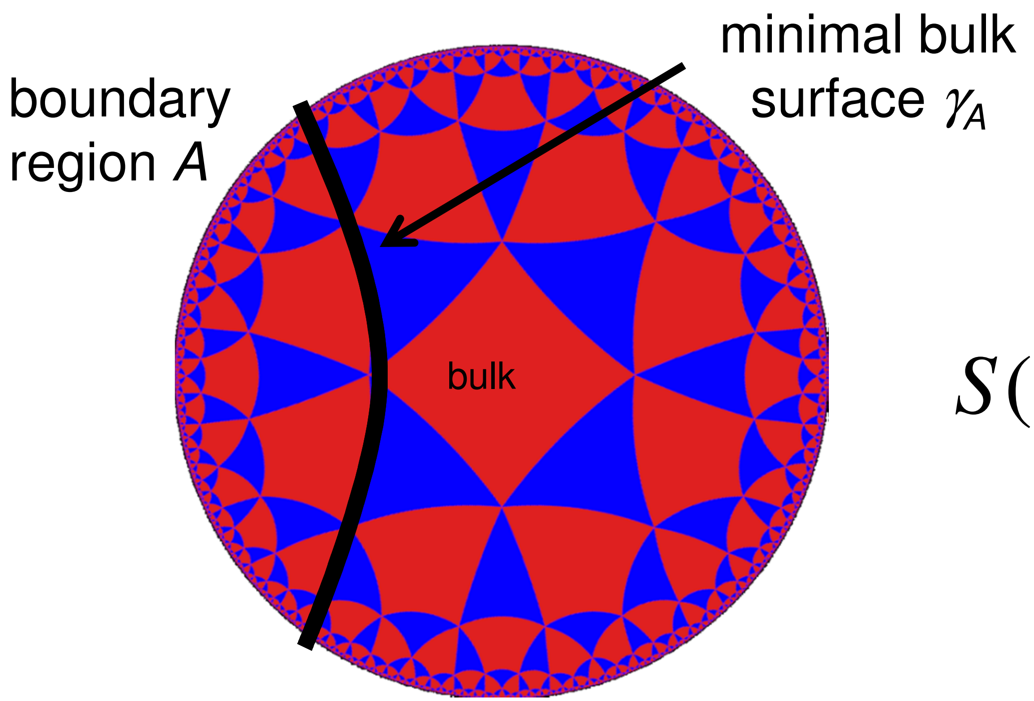
-- Emergent radial dimension can be regarded as an RG scale.

--Semiclassical (sub-AdS scale) bulk locality is highly nontrivial.

-- Geometry in the bulk theory is related to entanglement structure of the boundary theory.



Holographic entanglement entropy



$$S(A) = \frac{1}{4G_N}\mathrm{Area}(\gamma_A) + \cdots$$

To compute entropy of region A in the boundary field theory, find minimal area of the bulk surface γ_A with the same boundary (*Ryu-Takayanagi 2006*).

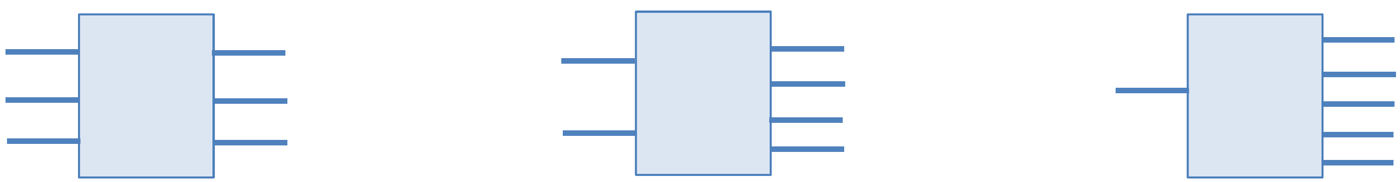
Perfect tensors

The tensor T arises in the expansion of a pure state of $2n$ v-dimensional "spins" in an orthonormal basis.

$$\boxed{|\psi\rangle = \sum_{a_1,a_2,\ldots,a_{2n}} T_{a_1a_2\ldots a_{2n}} \, |a_1a_2\ldots a_{2n}\rangle}$$

T is perfect if the state is maximally entangled across *any* cut, i.e. for any partition of the $2n$ spins into two sets of n spins. (State is *absolutely maximally entangled*.)

By transforming kets to bras, T also defines 3→3 unitary, 2→4 and 1→5 isometries.



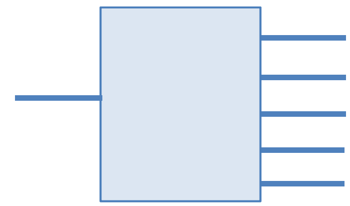
$$\sum_{a_1,\ldots a_6} T_{a_1\ldots a_6} \, |a_4a_5a_6\rangle\langle a_1a_2a_3| \qquad \sum_{a_1,\ldots a_6} T_{a_1\ldots a_6} \, |a_3a_4a_5a_6\rangle\langle a_1a_2| \qquad \sum_{a_1,\ldots a_6} T_{a_1\ldots a_6} \, |a_2a_3a_4a_5a_6\rangle\langle a_1|$$

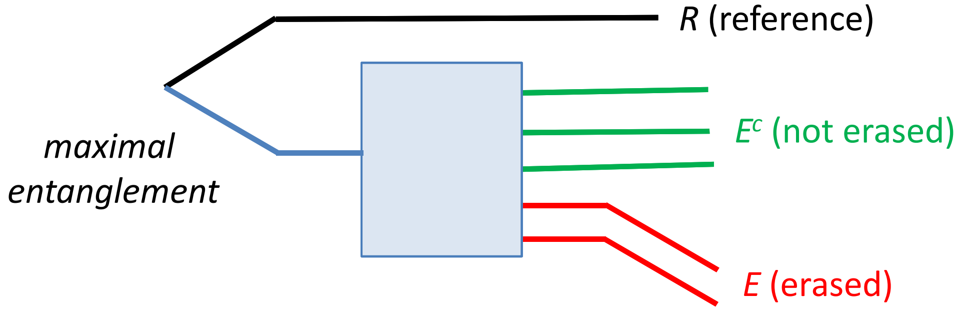
These are the isometric encoding maps (up to normalization) of quantum error-correcting codes. The 2→4 map encodes two qubits in a block of 4, and corrects 1 erasure. The 1→5 map encodes one qubit in a block of 5, and corrects 2 erasures.

Erasure correction

The 1$\rightarrow$5 isometric map encodes one qubit in a block of 5, and corrects two erasures.

$$\sum_{a_1,\ldots a_6} T_{a_1\ldots a_6} \,|\, a_2 a_3 a_4 a_5 a_6 \rangle\langle a_1 \,|$$





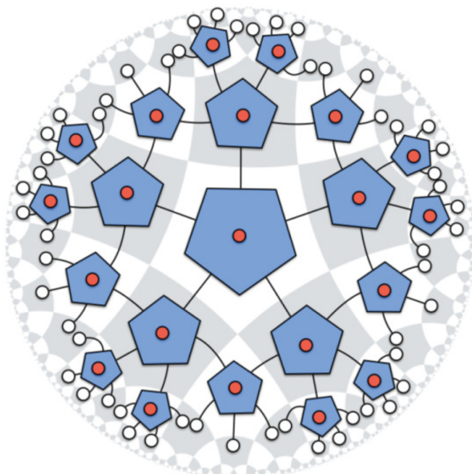
We say qubits are erased if they are removed from the code block. But we know *which* qubits were erased and may use that information in recovering from the error.

Consider maximally entangling a *reference qubit R* with the encoded qubit. Suppose two physical qubits (the subsystem *E*) are removed, while their complement E^c is retained.

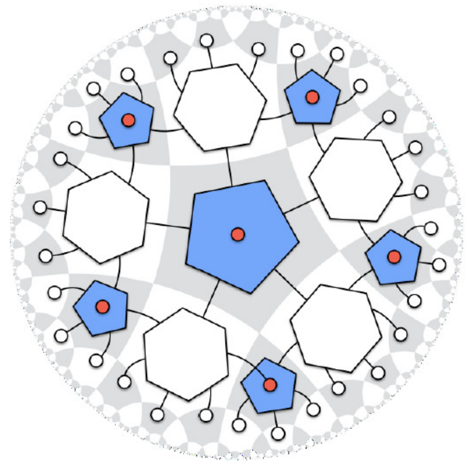
Because the tensor *T* is perfect, *RE* is maximally entangled with E^c, hence *R* is maximally entangled with a subsystem of E^c. Thus the logical qubit can be decoded by applying a unitary decoding map to E^c alone; *E* is not needed.

Likewise, we may apply any logical operator to the encoded qubit by acting on E^c alone. (The logical operation can be *cleaned* so it has no support on the erased qubits.)

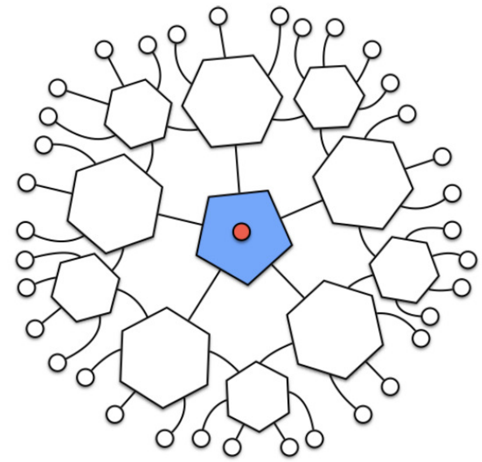
Holographic quantum codes



pentagon code



pentagon/hexagon code



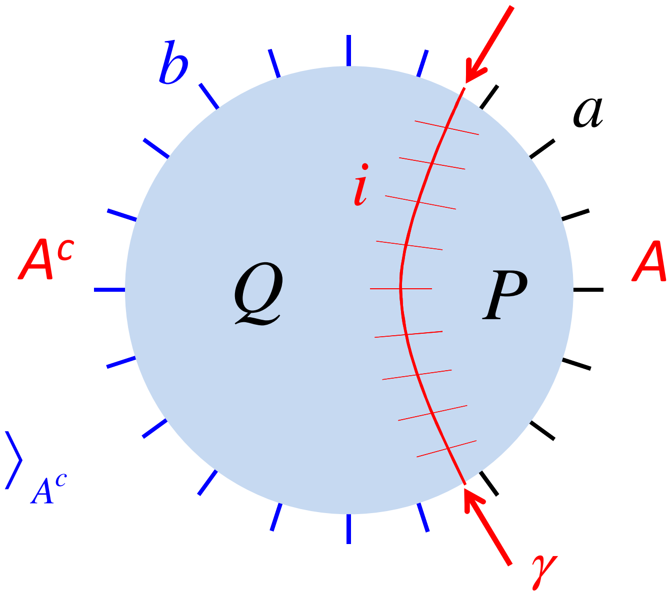
one encoded qubit

Holographic quantum error-correcting codes are constructed by contracting perfect tensors according to a tiling of hyperbolic space by polygons.

The code is an isometric embedding of the bulk Hilbert space into the boundary Hilbert space, obtained by composing the isometries associated with each perfect tensor.

Ryu-Takayanagi Formula

Consider a *holographic state* $|\psi\rangle$ (no dangling bulk indices), and a geodesic cut γ_A through the bulk with indices on the cut labeled by i. Indices of A are labeled by a and indices of A^c labeled by b.



$$|\psi\rangle = \sum_{a,b,i} |a\rangle_A \otimes |b\rangle_{A^c} P_{ai}\ Q_{bi} = \sum_i |P_i\rangle_A \otimes |Q_i\rangle_{A^c}$$

For a holographic state on a tiling with *nonpositive curvature*, the tensors P and Q are both *isometries*, if A is connected (*max-flow min-cut argument*).

If each internal index takes v values, there are $v^{|\gamma|}$ terms in the sum over i. and the vectors $\{|P\rangle_i\}$, $\{|Q\rangle_i\}$ are orthonormal. Therefore

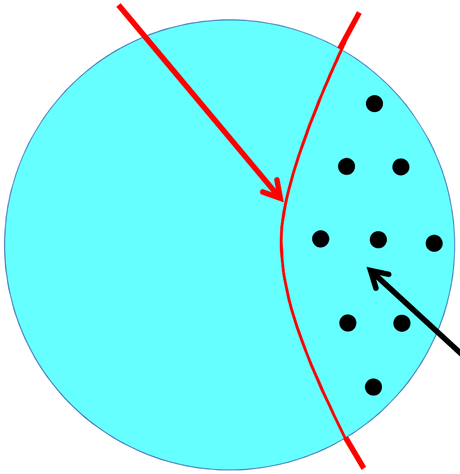
$$S(A) = |\gamma_A| \log v$$

Protection against erasure

For a connected region *A* on the boundary there is a corresponding *geodesic* γ_A . Bulk operators in the wedge between *A* and γ_A can be reconstructed on the boundary in region *A*.

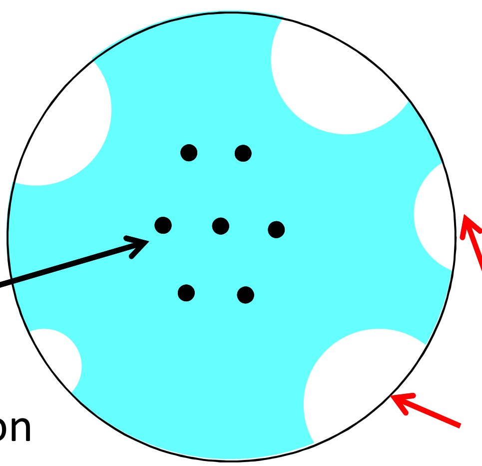
Operators deeper in the bulk have better protection against erasure on the boundary.

geodesic γ_A



A

Bulk operators reconstructable on boundary region A



Bulk operators reconstructable on unerased boundary

erased regions on boundary

Bulk operators at the center of the bulk are robust against erasure of up to half of the boundary qubits.

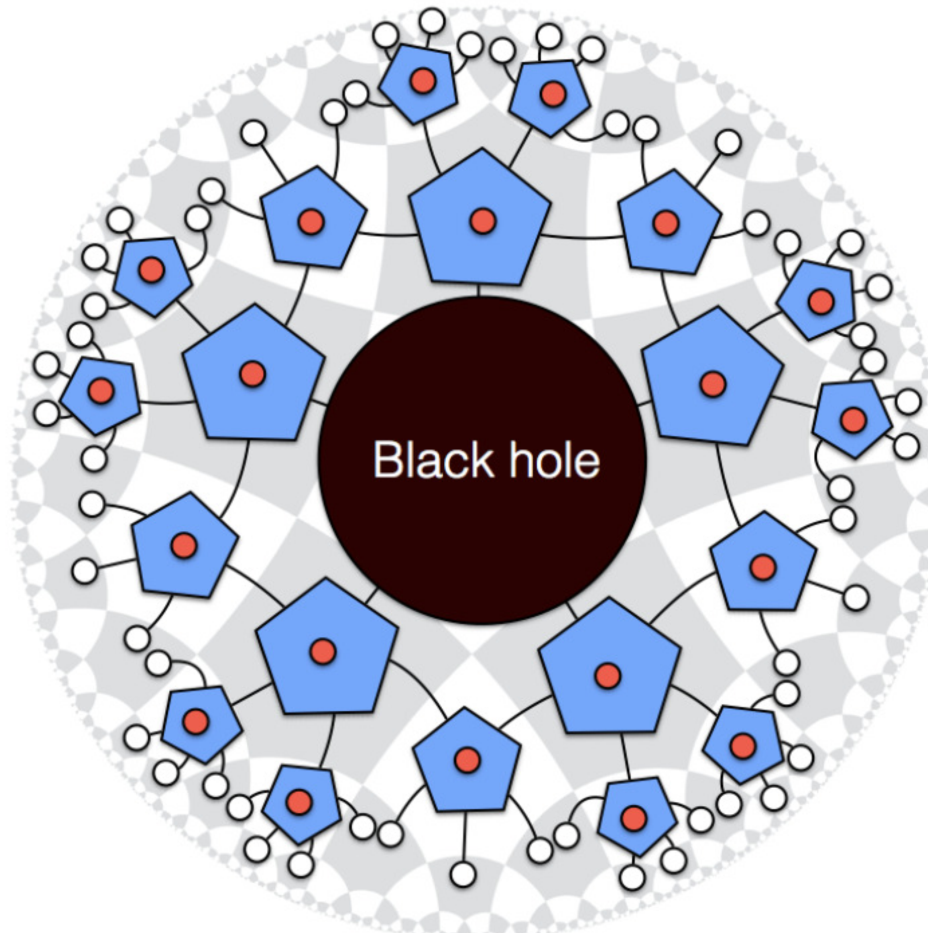
Holographic black holes

-- Most boundary states correspond to large black holes in the bulk.

-- Bulk local operators acting outside the black hole can be reconstructed on the boundary.

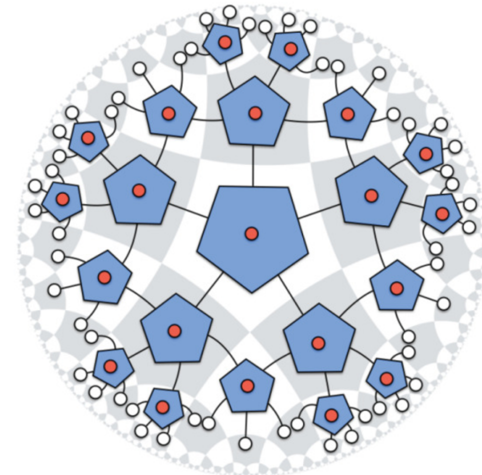
-- Uncontracted bulk indices at the horizon, the black hole microstates, are also mapped to the boundary.

-- Encoding isometry becomes trivial as black hole grows to fill the whole bulk.

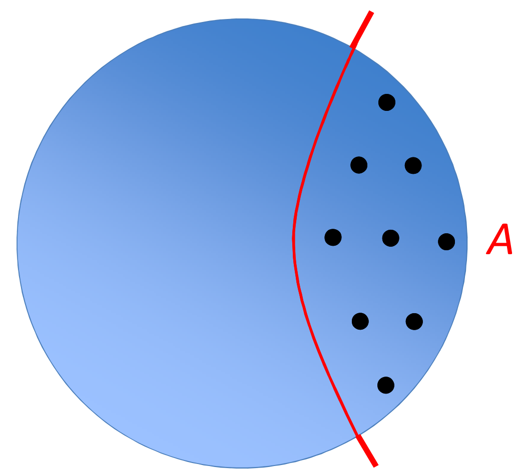


Holographic quantum codes

-- Nicely capture some central features of full blown gauge/gravity duality, and provide an explicit dictionary relating bulk and boundary observables.



-- Realize exactly the Ryu-Takayanagi relation between boundary entanglement and bulk geometry (with small corrections in some cases).



-- But … so far these models are not dynamical, and do not address bulk locality at sub-AdS distance scales.

Quantumists ≈ Biologists

quantum gravity = life

boundary theory = chemistry

quantum information theorists = chemists

quantum gravity theorists = biologists

what we want = molecular biology

black hole information problem = fruit fly

understanding the big bang = curing cancer

Slide concept stolen from Juan Maldacena

Ooguri: I see that this new joint activity between quantum gravity and quantum information theory has become very exciting. Clearly entanglement must have something to say about the emergence of spacetime in this context.

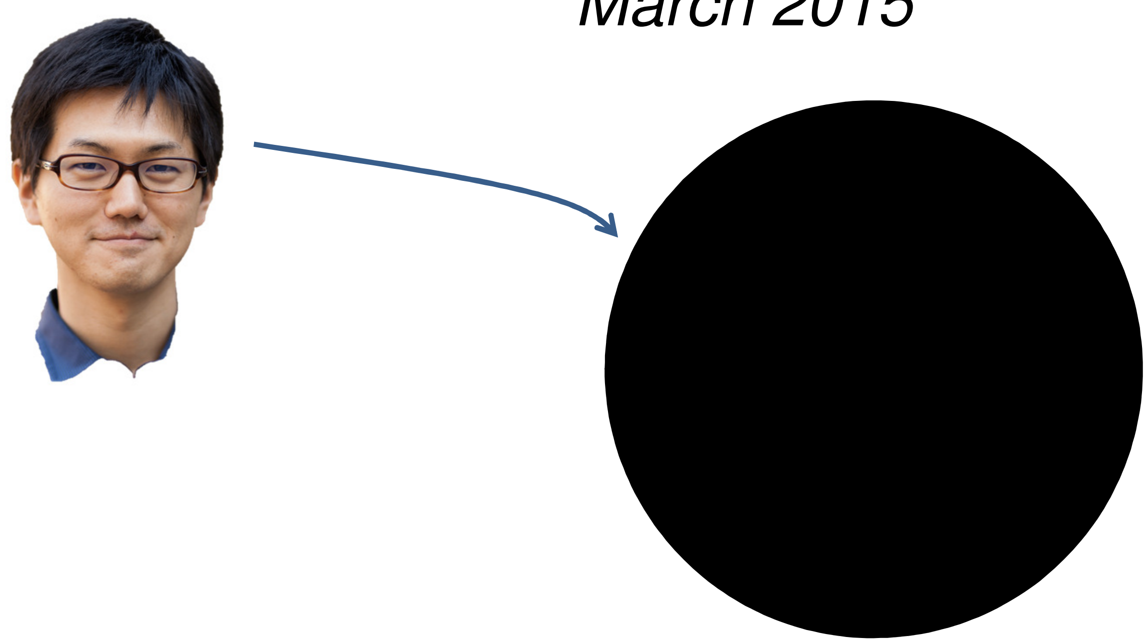
Witten: I hope so. I'm afraid it's hard to work on, so in fact I've worked with more familiar kinds of questions.

Kavli IPMU News
December 2014

Notices of AMS
May 2015

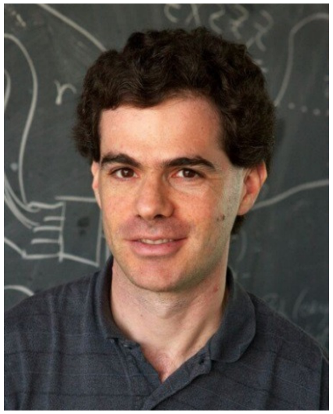
“Now is the time for quantum information scientists to jump into .. black holes”

Beni Yoshida
QuantumFrontiers.com
March 2015



Deep insights into the quantum structure of spacetime will arise from laboratory experiments studying highly entangled quantum systems.

Daniel Gottesman,
Fault tolerance in small experiments



David DiVincenzo,
Quantum error correction and the future of solid state quantum computing

Philipp Schindler,
Quantum error correction with trapped ions

Michel Devoret,
Quantum error correction in superconducting circuits