Semilocal defects

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I analyze the interplay of gauge and global symmetries in the theory of topological defects. In a two-dimensional model in which both gauge symmetries and *exact* global symmetries are spontaneously broken, stable vortices may fail to exist even though magnetic flux is topologically conserved. Following Vachaspati and Achúcarro, I formulate the condition that must be satisfied by the pattern of symmetry breakdown for finite-energy configurations to exist in which the conserved magnetic flux is spread out instead of confined to a localized vortex. If this condition is met, vortices are always unstable at sufficiently weak gauge coupling. I also describe the properties of defects in models with an "accidental" symmetry that is partially broken by gauge-boson exchange. In some cases, the spontaneously broken accidental symmetry is not restored inside the core of the defect. Then the structure of the defect can be analyzed using an effective field theory; the details of the physics responsible for the spontaneous symmetry breakdown need not be considered. Examples include domain walls and vortices that are classically unstable, but are stabilized by loop corrections, and magnetic monopoles that have an unusual core structure. Finally, I examine the general theory of the "electroweak strings" that were recently discussed by Vachaspati. These arise only in models with gauge-boson "mixing," and can always end on magnetic monopoles. Cosmological implications are briefly discussed.

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I. INTRODUCTION

A gauge theory that undergoes the Higgs mechanism will in many cases contain topologically stable defects [1,2]. For example, in two spatial dimensions, the classical field configurations may be classified by a conserved magnetic flux, such that there are infinite energy barriers separating configurations with different values of the magnetic flux. The configuration of minimum energy in at least one of the nontrivial magnetic flux sectors is then expected to be a localized vortex with magnetic flux trapped in its core, a static soliton solution to the classical field equations. When the theory is quantized, the vortex survives as a stable particle in the spectrum. The corresponding defect in three spatial dimensions is a one-dimensional string.

But it was recently noted by Vachaspati and Achúcarro [3] that, even if magnetic flux is topologically conserved, and a finite-energy gap separates the nontrivial flux sectors from the vacuum sector, there may be no stable vortex solutions. This can happen if, in addition to the spontaneously broken gauge symmetry, there is also a spontaneously broken *exact* global symmetry, and so exactly massless Nambu-Goldstone bosons in the spectrum. The nontrivial magnetic flux sectors may then contain configurations of finite energy in which the magnetic flux is spread out over an arbitrarily large area, and it becomes a dynamical question whether the energy is minimized by the localized vortex or the configuration with unlocalized magnetic flux. Vortices that are potentially subject to this instability were called "semilocal" in Ref. [3], in recognition of the important role played by the global symmetry.

The purpose of this paper is to give a systematic account of the interplay of gauge and global symmetries in the classification of topologically stable defects, in a more general setting than considered in Ref. [3]. I will formulate the criterion for the existence of finite energy configurations that carry a topologically conserved magnetic flux that is unlocalized, and will note the existence of both vortices and domain walls that are classically unstable, but are stabilized by quantum effects involving gauge boson loops. I will also discuss "semilocal monopoles" that, while always classically stable, can have a different kind of core structure than the usual gauge theory monopoles. Finally, I discuss some general properties of "electroweak vortices," which are classically stable even though they carry no topologically conserved flux [4].

Throughout this paper, the term "semilocal" has a more general meaning than in Ref. [3]. In the original usage of Ref. [3], semilocal defects arise in models in which gauge symmetries "mix" with exact global symmetries. I extend this usage to encompass models with approximate global symmetries as well. The original notion might be called "strict semilocality" and the extended version "generalized semilocality." In practice, though, the context will typically leave no ambiguity about what sense of the term is meant, and no modifier will be needed.

The general approach adopted here is especially suitable for models in which gauge or global symmetries are *dynamically* broken, or for any scheme in which it is convenient to "integrate out" the detailed physics responsible for the symmetry breaking. Assuming that the relevant gauge couplings are weak, the semilocal defects discussed here can be studied using an effective field

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theory in which only light degrees of freedom are retained. The typical size of the defects is larger than the distance scale associated with the symmetry breakdown by an inverse power of the weak gauge coupling. The distinguishing feature of "semilocal" defects, then, is that their detailed structure can be analyzed without ever considering the "restoration" of the spontaneously broken symmetry.

In Sec. II, I describe the general class of models that will be considered in this paper. These models have an "accidental" symmetry, and part of this symmetry is gauged. The accidental symmetry is spontaneously broken. To determine the unbroken gauge group, we need to solve a "vacuum alignment" problem.

The general theory of semilocal vortices is discussed in Sec. III. Two cases are considered. In the first case, there are exactly massless Nambu-Goldstone bosons, and the topologically conserved magnetic flux need not be confined. Stable vortices may exist for a range of values of the gauge coupling, but vortices become unstable when the gauge coupling is sufficiently weak. In the second case, there are light "pseudo Goldstone" bosons; vortices become classically unstable at weak gauge coupling, but are stabilized by quantum corrections. Then the accidental symmetry is not restored inside the core of the vortex.

Section IV concerns semilocal domain walls and monopoles. These are always stable. They resemble the semilocal vortices in the second case above; the core of the defect has an unusual structure, because the accidental symmetry is not restored inside the core.

Examples that illustrate the general theory are presented in Secs. V and IV. The models in Sec. V have elementary Higgs fields. The models in Sec. VI do not; instead, the spontaneous symmetry breakdown is dynamical.

The criterion for the existence of configurations with unconfined magnetic flux is further discussed in Sec. VII. I show that finite-energy configurations can exist in which the conserved magnetic flux is "spread out" only if gauge and global symmetries "mix;" the unbroken global symmetry group must have generators that are nontrivial linear combinations of spontaneously broken gauge symmetry generators and global symmetry generators.

The general theory of electroweak vortices [4] is described in Sec. VIII. These carry no conserved magnetic flux, yet are classically stable. Their distinguishing feature is that they become stable semilocal vortices in the limit in which some gauge coupling approaches zero. This is possible only if the pattern of gauge symmetry breaking admits gauge-boson mixing. (In other words, there are unbroken gauge generators that are nontrivial linear combinations of generators that belong to distinct invariant subalgebras of the gauged Lie algebra.) I note that electroweak strings can end on magnetic monopoles, and compute the magnetic charge of the monopole. I also discuss the Aharonov-Bohm interactions of electroweak strings, and point out that an electroweak string cannot be used to detect the "quantum hair" of an object. Finally, I comment on the "embedded defects" recently discussed by Vachaspati and Barriola [5] and remark that embedded monopoles are always unstable.

Section IX contains some concluding remarks, including comments on the implications of electroweak strings for particle physics and cosmology.

II. GENERAL FORMALISM

I will consider a class of gauge theories that can be characterized as follows [6]. In the limit of vanishing gauge couplings, the theory respects a group $G_{\rm approx}$ of global symmetries that is spontaneously broken to the subgroup $H_{\rm approx}$.[1] ($G_{\rm approx}$ is a finite-dimensional compact Lie group that we will assume is connected.) In this limit, the theory has a degenerate vacuum manifold, and massless Nambu-Goldstone bosons, characterized by the coset space $G_{\rm approx}/H_{\rm approx}$.

Now suppose that a subgroup $G_{\rm gauge}$ of $G_{\rm approx}$ is coupled to gauge fields. The gauging intrinsically breaks the $G_{\rm approx}$ symmetry and partially lifts the vacuum degeneracy. The surviving exact symmetry group is the subgroup of $G_{\rm approx}$ that preserves the embedding of $G_{\rm gauge}$ in $G_{\rm approx}$; that is,

$$G_{\rm exact}=\{g\in G_{\rm approx}|gG_{\rm gauge}g^{-1}=G_{\rm gauge}\}\ . \tag{2.1}$$

Since $G_{\rm gauge}$ is an invariant subgroup of $G_{\rm exact}$, and $G_{\rm exact}$ is compact, $G_{\rm exact}$ has the local structure $G_{\rm exact}\sim G_{\rm gauge}\times G_{\rm exact}^{\rm global}$, but it may also include discrete automorphisms of $G_{\rm gauge}$; these will be relevant to the discussion of domain walls below.

The unbroken gauge group $H_{\rm gauge}$ is the intersection $G_{\rm gauge}$ with $H_{\rm approx}$, and the unbroken exact symmetry group $H_{\rm exact}$ is the intersection of $G_{\rm exact}$ with $H_{\rm approx}$. However, these unbroken groups cannot be determined by group theory alone. There is, in general, a nontrivial issue of "vacuum alignment" that must be resolved by the dynamics of the theory [6]. If we fix the embedding of $G_{\rm gauge}$ in $G_{\rm approx}$, then these intersections depend on how $H_{\rm approx}$ is embedded in $G_{\rm approx}$; in other words, the unbroken groups depend on how the vacuum is chosen from the (approximate) vacuum manifold $G_{\rm approx}/H_{\rm approx}$. The gauge interactions lift the degeneracy of the approximate vacuum states, and determine the alignment. The lifting of the degeneracy is a quantum effect arising from gauge-boson loops.

Once the alignment is determined, we can divide the $G_{\rm approx}/H_{\rm approx}$ Nambu-Goldstone bosons into three classes. The $G_{\rm gauge}/H_{\rm gauge}$ bosons are absorbed by gauge fields. The $G_{\rm exact}/H_{\rm exact}$ bosons that are not $G_{\rm gauge}/H_{\rm gauge}$ bosons remain exactly massless. And the $G_{\rm approx}/H_{\rm approx}$ bosons that are not $G_{\rm exact}/H_{\rm exact}$ bosons acquire nonzero masses due to the gauge interactions; they are "pseudo Goldstone" bosons [7].

Though it will often be convenient to think of the breakdown of $G_{\rm approx}$ to $H_{\rm approx}$ as due to the condensation of an elementary Higgs scalar, the above discussion makes no assumption about the mechanism of the symmetry breakdown. In particular, it applies to the case of

[1] I use this notation because the $G_{\rm approx}$ symmetry will typically be broken when the gauge interactions turn on.

a theory that contains no elementary scalars at all, in which the condensate is a composite operator bilinear in elementary fermions, as in technicolor models [6,8].

The symmetry-breaking scheme outlined here sometimes suffers from the flaw of "unnaturalness," or the need to fine-tune bare parameters. For example, in a theory with elementary scalars, it may be that the most general Higgs potential of renormalizable type (a quartic polynomial in the Higgs field) that is invariant under the $G_{\rm exact}$ symmetry is not also invariant under the larger $G_{\rm approx}$ symmetry. Then radiative corrections will induce divergent symmetry-breaking terms in the potential that must be removed with suitable counterterms. This scheme is unnatural in the sense that the feature that the $G_{\rm approx}$ symmetry is broken *only* by radiative corrections (and not by terms in the classical Higgs potential) results from a delicate cancellation between bare parameters and radiatively induced renormalization of parameters.

This naturalness problem is typically avoided in models without elementary scalars, and sometimes in other cases as well. Examples will be discussed in Secs. V and VI.

III. VORTICES

Given the pattern of symmetry breakdown described above, let us classify the nonsingular classical field configurations that have finite energy, in two spatial dimensions [1]. For the Higgs field potential energy to be finite, the Higgs field must reside in the exact vacuum manifold $G_{\rm exact}/H_{\rm exact}$ on the circle at $r=\infty$. For the Higgs field gradient energy to be finite, the Higgs field must be covariantly constant on the circle at $r=\infty$.

Since $G_{\rm exact}$ acts transitively on the exact vacuum manifold (assuming no exact "accidentally degeneracy"), we may perform a $G_{\rm exact}$ transformation that rotates the Higgs field at the point $(r=\infty,\theta=0)$ to a standard value Φ_0. Since it is covariantly constant, the Higgs field on the circle at infinite must lie in the orbit of the gauge group acting on Φ_0; it can be expressed as

$$\begin{aligned}&\Phi(r=\infty,\theta)=D[g(\theta)]\Phi_0,\quad g(\theta)\in G_{\rm gauge}\ ,\\ &g(0)=e,\quad g(2\pi)\in H_{\rm gauge}\ ,\end{aligned}\tag{3.1}$$

where D is the representation of $G_{\rm gauge}$ according to which Φ transforms. Equation (3.1) associates with each finite energy field configuration a closed path in the coset space $G_{\rm gauge}/H_{\rm gauge}$ that begins and ends at the trivial coset. Thus, the nonsingular field configurations of finite energy can be classified by the fundamental group $\pi_1(G_{\rm gauge}/H_{\rm gauge})$.[2] There is an infinite energy barrier separating configurations that correspond to different elements of this group, while configurations that correspond to the same element can be smoothly deformed one to another, while the energy remains finite.

[2]Actually, there is an ambiguity in this correspondence when $H_{\rm gauge}$ is non-Abelian and disconnected. This ambiguity can be resolved if we consider patching together distantly separated configurations; it has no effect on the ensuing discussion.

Because $G_{\rm gauge}\subseteq G_{\rm exact}\subseteq G_{\rm approx}$, we have the inclusion $G_{\rm gauge}/H_{\rm gauge}\subseteq G_{\rm exact}/H_{\rm exact}\subseteq G_{\rm approx}/H_{\rm approx}$ (for the coset spaces are obtained by the action of the groups on Φ_0). Thus, there are natural homomorphisms:

$$\begin{aligned}&\pi_1(G_{\rm gauge}/H_{\rm gauge})\to\pi_1(G_{\rm exact}/H_{\rm exact})\ ,\\ &\pi_1(G_{\rm exact}/H_{\rm exact})\to\pi_1(G_{\rm approx}/H_{\rm approx})\ ;\end{aligned}\tag{3.2}$$

each loop in $G_{\rm gauge}/H_{\rm gauge}$ is also a loop in $G_{\rm exact}/H_{\rm exact}$ and each loop in $G_{\rm exact}/H_{\rm exact}$ is also a loop in $G_{\rm approx}/H_{\rm approx}$.

We can distinguish three types of elements of $\pi_1(G_{\rm gauge}/H_{\rm gauge})$, according to whether the element belongs to the kernel of these homomorphisms. First consider an element that is not in the kernel of either homomorphism. This means that the corresponding noncontractible loop in the gauged vacuum manifold $G_{\rm gauge}/H_{\rm gauge}$ remains noncontractible in the (larger) approximate vacuum manifold. Hence, the finite-energy field configurations associated with this loop cannot lie in the approximate vacuum manifold everywhere. Each field configuration must therefore have a "core" somewhere where the Higgs field potential energy density is nonvanishing. If we minimize the energy in this topological sector, we will obtain a static vortex solution to the classical field equations, or perhaps a configuration of two or more widely separated vortices.

Second, consider a nontrivial element of $\pi_1(G_{\rm gauge}/H_{\rm gauge})$ that is in the kernel of the first homomorphism. This means that the corresponding noncontractible loop in $G_{\rm gauge}/H_{\rm gauge}$ can be contracted in the exact vacuum manifold $G_{\rm exact}/H_{\rm exact}$. Hence, we can construct finite-energy configurations in this class that live in the exact vacuum manifold everywhere, and have no Higgs field potential energy.

(Because $G_{\rm exact}$ has the local structure $G_{\rm exact}\sim G_{\rm gauge}\times G_{\rm exact}^{\rm global}$, this kernel can be nontrivial only if there is mixing of gauge and global symmetries. That is, $H_{\rm exact}$ must be nontrivial, and there must be $H_{\rm exact}$ generators that are linear combinations of $G_{\rm gauge}$ generators and $G_{\rm exact}^{\rm global}$ generators. I will discuss this point further in Sec. VII.)

To understand these configurations better, consider the classical field theory in the limit of *infinite* gauge coupling. Then the gauge field is nondynamical—gauge fields carry no energy. Still the gauging has nontrivial consequences, for Higgs field configurations that differ by a gauge transformation are effectively identified. The *physical* vacuum manifold is not $G_{\rm exact}/H_{\rm exact}$, but rather this coset space with the action of the gauge group $G_{\rm gauge}$ modded out. That is, it is the space $M_{\rm orbit}$ of $G_{\rm gauge}$ orbits on $G_{\rm exact}/H_{\rm exact}$.

In this limit, the configurations such that the Higgs field lies in the exact vacuum manifold everywhere have only gradient energy. And gradient energy in two spatial dimensions is scale invariant. Thus, if we find the configuration of this type that has minimal energy, there will actually be an infinite set of such configurations, parametrized by an arbitrary size scale. What we have constructed is a two-dimensional "Skyrmion" [9] (or "global texture" [10]) associated with a topologically nontrivial mapping from the two-sphere (the plane plus the point at infinity) to the physical vacuum manifold $M_{\rm orbit}$. Its energy will be

$$m_{\rm Skyrmion} = Cv^2 , \tag{3.3}$$

where v is the symmetry breaking scale and C is a numerical constant of order 1, for v is the only relevant scale.

Now let us reintroduce the gauge field kinetic term. The Skyrmions that we have constructed carry nonzero magnetic flux. (The gauge field cannot be a pure gauge everywhere, because it is topologically nontrivial on the circle at infinity, and is smooth on the plane.) When the gauge field dynamics turns on, this flux will want to spread out. The Skyrmion of infinite size now will have the lowest energy; in fact, its gauge field energy will vanish.

What we have found, then, is that in a sector whose "magnetic flux" is characterized by a noncontractible loop in $G_{\rm gauge}/H_{\rm gauge}$ that can be contracted in $G_{\rm exact}/H_{\rm exact}$, configurations of finite energy can be constructed such that the flux is spread out over an arbitrarily large area. This sector is separated from sectors with other values of the flux by an infinite energy barrier. But within this sector there are configurations in which the energy density is arbitrarily small everywhere (although the total energy is bounded from below by Cv^2). Notice that this is possible only in a theory that contains exactly massless Nambu-Goldstone bosons, for only then can a scale-invariant Skyrmion exist.

In a magnetic flux sector of this type, there will of course also be configurations in which the magnetic flux is trapped inside a vortex core where the Higgs field leaves the exact vacuum manifold. It becomes a dynamical question (not a topological one) whether the vortex configurations or the spread-out configurations have lower energy. In the limit of large gauge coupling, the vortex energy is

$$m_{\rm vortex} = C'v^2 , \tag{3.4}$$

where C' is a numerical constant of order one, independent of coupling constants. [The leading contribution to the vortex energy that depends on the Higgs potential is of order $(\lambda/e^2)v^2$, where λ is a scalar self-coupling and e is the gauge coupling; it can be neglected in the limit $e^2 \to \infty$.] The vortex is stable, in this limit, if $C' < C$. Whether this is the case depends on the detailed geometry of the vacuum manifold. But a definite statement can be made about the opposite limit of weak gauge coupling. In this limit, the vortex carries enormous magnetic flux that must spread out. Any configuration with a Higgs field core that remains bounded in this limit carries an energy that scales like

$$m_{\rm vortex} \sim v^2 \ln(1/e^2) . \tag{3.5}$$

This behavior results from the competition between Higgs field gradient energy of order $v^2\ln(r)$ and magnetic field energy of order $1/(e^2r^2)$, where r is the size of the region occupied by the flux. Thus, when the gauge coupling is sufficiently weak, the Skyrmion configuration minimizes the energy, and there is no stable vortex in this flux sector.

Even if the Skyrmion minimizes the energy in a magnetic flux sector, there may be a vortex configuration (with finite core size) in the same sector that is classically stable. The vortex will then be metastable and will decay via quantum tunneling. From the Euclidean path integral viewpoint, the instanton configuration that mediates the decay is a "global monopole [11]." In the limit of infinite gauge coupling, this is a configuration with a nontrivial Higgs field core, where the Higgs field on a large sphere surrounding the core assumes the nontrivial mapping from the two-sphere to the exact vacuum manifold that is associated with the Skyrmion. For finite gauge field coupling, this configuration has magnetic flux that enters the core from a narrow tube (the vortex) and then spreads out and returns to infinity (the Skyrmion). Similarly, a string in three spatial dimensions is metastable for this range of parameters, because the string can break by nucleating a global monopole-antimonopole pair. The long-range interaction energy between a pair of global monopoles with separation r is Cv^2r [with C defined by Eq. (3.3)], so it is energetically favorable for the monopole pair to form if the string tension is greater than Cv^2. These decay processes are further discussed in Ref. [12].

Finally, consider an element of $\pi_1(G_{\rm gauge}/H_{\rm gauge})$ that is in the kernel of the second homomorphism in Eq. (3.2) but not the first. This means that the corresponding noncontractible loop in $G_{\rm gauge}/H_{\rm gauge}$ remains noncontractible in $G_{\rm exact}/H_{\rm exact}$, but can be contracted in $G_{\rm approx}/H_{\rm approx}$. Hence, we can construct configurations in this flux sector such that the Higgs field lies in the approximation vacuum manifold everywhere, but not configurations that lie in the exact vacuum manifold everywhere. When the gauge coupling is sufficiently weak, the vortex solutions become classically unstable, and the flux wants to spread out. But quantum corrections due to gauge-boson exchange prevent the vortex from spreading to infinity.

IV. DOMAIN WALLS AND MONOPOLES

Within the symmetry breaking scheme formulated in Sec. II, we may also consider the properties of topological domain walls and monopoles. Though there are no unexpected instabilities, these defects can have some unusual properties that are worthy of note.

A. Domain walls

The nonsingular configurations that have finite energy in one spatial dimension are classified by the group $\pi_0(G_{\rm exact}/H_{\rm exact})$.[3] For the Higgs field potential energy to be finite, the Higgs field must take a value in the exact vacuum manifold $G_{\rm exact}/H_{\rm exact}$ at both points at infinity. By performing a suitable $G_{\rm exact}$ transformation, we may choose the Higgs field at $x = -\infty$ to assume the standard value Φ_0. Two configurations can be smoothly deformed

[3]We need not be concerned with the ambiguity in this classification that can arise when $H_{\rm exact}$ is non-Abelian.

one to the other while the energy remains finite if and only if $\Phi(x=\infty)$ for both configurations lies in the same connected component of the exact vacuum manifold. By minimizing the energy in a nontrivial sector, we construct a static domain wall solution to the classical field equations (or perhaps two or more distantly separated domain walls).

A nontrivial element of $\pi_0(G_{\text{exact}}/H_{\text{exact}})$ may be in the kernel of the homomorphism

$$\pi_0(G_{\text{exact}}/H_{\text{exact}}) \rightarrow \pi_0(G_{\text{approx}}/H_{\text{approx}}) ; \tag{4.1}$$

that is, a vacuum state that is not connected to Φ_0 in the exact vacuum manifold may be connected to Φ_0 in the approximate vacuum manifold. Then the domain wall will be classically unstable. It can be deformed to a configuration that has no classical Higgs potential energy, and it will then want to spread out to minimize its gradient energy. But the quantum corrections to the effective Higgs potential, generated by gauge-boson exchange, will prevent the domain wall from spreading indefinitely, and will stabilize it.

B. Monopoles

In order that a field configuration have finite energy in three spatial dimensions, the Higgs field must take values in $G_{\text{exact}}/H_{\text{exact}}$ on the two-sphere at $r=\infty$, and must be covariantly constant on the two-sphere. Thus, nonsingular finite-energy configurations are classified by [1] $\pi_2(G_{\text{gauge}}/H_{\text{gauge}})=\pi_1(H_{\text{gauge}})/\pi_1(G_{\text{gauge}})$;[4] they are associated with noncontractible closed paths in H_{gauge}, beginning and ending at the identity, that are contractible in G_{gauge}. The element of $\pi_1(H_{\text{gauge}})$ associated with a nontrivial sector identifies the topologically conserved magnetic charge of that sector [13,1].

The homomorphism

$$\pi_2(G_{\text{gauge}}/H_{\text{gauge}}) \rightarrow \pi_2(G_{\text{exact}}/H_{\text{exact}}) \tag{4.2}$$

has a trivial kernel. This is because H_{exact} has the general form $H_{\text{exact}}=(H_{\text{exact}}\times H^{\text{global}}_{\text{exact}})/H_{\text{discrete}}$, where H_{discrete} is a discrete invariant subgroup of $H_{\text{gauge}}\times H^{\text{global}}_{\text{exact}}$. Thus, a loop that is noncontractible in H_{gauge} remains noncontractible in H_{exact}. We conclude that there are no "semilocal monopoles" that are precisely analogous to the semilocal vortices considered in Ref. [3]—in a configuration with nonzero magnetic charge, the Higgs field cannot lie in the *exact* vacuum manifold everywhere.

But there can be nontrivial magnetic charge sectors that contain configurations such that the Higgs field lies in the *approximate* vacuum manifold everywhere. These sectors are associated with noncontractible loops in H_{gauge} that are contractible in H_{approx}, or in other words, with the kernel of the homomorphism

$$\pi_2(G_{\text{gauge}}/H_{\text{gauge}}) \rightarrow \pi_2(G_{\text{approx}}/H_{\text{approx}}) . \tag{4.3}$$

Ignoring quantum effects, these monopole configurations have only gradient energy and magnetic field energy. The gradient energy makes them want to shrink, but they are prevented from collapsing completely by their magnetic field energy.

These "semilocal" magnetic monopoles have a different core structure than the usual gauge theory monopoles.[5] "Heavy" broken gauge fields are excited in the core, and the embedding of G_{gauge} in G_{approx} varies in the core, but the spontaneously broken G_{approx} symmetry is not "restored" anywhere. It is a dynamical question, depending on the details of the Higgs potential, whether the realization of the G_{approx} symmetry actually changes inside the core of the monopole configuration with minimal energy.

I should clarify the difference between semilocal monopoles and the monopoles that arise in typical grand unified theories. It is a general feature, shared by semilocal monopoles and monopoles of the usual kind, that the realization of the *gauge* symmetry must be different inside the monopole core than in the vacuum (at least at an isolated point inside the core). This is not to say that the gauge symmetry is fully restored inside the core. In the SU(5) model, for example, if we ignore the electroweak symmetry breakdown, a Higgs field in the adjoint representation breaks the gauge symmetry to $[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)]/Z_6$. Inside the core of the minimally charged magnetic monopole, the stability group of the Higgs field is reduced to a subgroup of the symmetry of the vacuum [15]; namely, $[\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{U}(1)]/[Z_6\times Z_2]$. At the center of the core, this symmetry is enhanced to $[\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)]/[Z_2\times Z_2]$.

This example illustrates the generic case. The symmetry H_{core} inside the core is a subgroup of the symmetry H_{gauge} in the vacuum. The topological magnetic charge of the monopole can be characterized by a noncontractible closed path in H_{core} that begins and ends at the identity. In order for the Higgs field to be smooth, this symmetry must enlarge at the center of the core to $H_{\text{center}}\supset H_{\text{core}}$, such that this closed path in H_{core} can be contracted in H_{center}. We see that H_{center} cannot be contained in H_{gauge}, but it is not necessary for H_{center} to contain H_{gauge}, either.

In a semilocal monopole, too, the subgroup H_{center} of G_{gauge} that preserves the Higgs field at the center of the core is not contained in H_{gauge}. But this is achieved even though the stability group of the Higgs field is H_{approx} everywhere. The realization of the gauge symmetry changes inside the core because the relative alignment of H_{approx} and G_{gauge} adjusts there. This means that the core of the monopole can be accurately described in an "effective field theory" that describes physics below the scale of the symmetry breakdown, as I will discuss in more detail in Sec. VI.

V. EXAMPLES

I will now apply the above discussion to a sample model. When all gauge interactions are turned off, the

[4]We need not be concerned with the ambiguity in this classification that can arise when H_{gauge} is disconnected and non-Abelian.

[5]Note that the term "semilocal monopole" is used differently here than in Refs. [3] and [14].

Lagrange density of this model is

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a-\frac{\lambda}{8}(\pi^a\pi^a-v^2)^2\ , \tag{5.1}$$

where $a=0,1,2,3$. Thus, in this limit, the symmetry breaking pattern of the model is

$$G_{\text{approx}}=\mathrm{SO}(4)\to H_{\text{approx}}=\mathrm{SO}(3)\ , \tag{5.2}$$

and there are three Nambu-Goldstone bosons, plus one massive Higgs field with a mass

$$m_S^2=\lambda v^2\ . \tag{5.3}$$

It is convenient to write the Higgs field as a two-by-two matrix

$$\Phi=\frac{1}{\sqrt{2}}(\pi^0+i\boldsymbol{\pi}\cdot\boldsymbol{\sigma})\equiv\begin{bmatrix}\phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^*\end{bmatrix}\ , \tag{5.4}$$

which transforms under $G_{\text{approx}}=[\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R]/Z_2$ according to

$$\Phi\to U_L\Phi U_R^\dagger\ . \tag{5.5}$$

A. Unstable vortex

Let us briefly recall the model that was analyzed in Ref. [3]. It is obtained by gauging the U(1) subgroup of one of the SU(2)'s. We choose to gauge the $\mathrm{U}(1)_R$ generated by

$$Y_R=\mathrm{diag}\left[\frac{1}{2},-\frac{1}{2}\right]\ . \tag{5.6}$$

Then the exact symmetry of the model is

$$G_{\text{exact}}=(\mathrm{SU}(2)_L\times[\mathrm{U}(1)_R\rtimes Z_{2,R}])/Z_2\ , \tag{5.7}$$

where $\rtimes$ denotes a semidirect product. Here the $Z_{2,R}$ is generated by the charge conjugation operation

$$\mathcal{C}_R:\phi\equiv\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}\to\phi^c=-i\sigma_2\phi^*=\begin{bmatrix}-\phi_2^*\\ \phi_{1*}\end{bmatrix}\ . \tag{5.8}$$

This operation commutes with $\mathrm{SU}(2)_L$, but anticommutes with Y_R:

$$\mathcal{C}_R Y_R\mathcal{C}_R^{-1}=-Y_R\ ; \tag{5.9}$$

it is a nontrivial automorphism of the $\mathrm{U}(1)_R$ gauge group. In this case, the G_{approx} symmetry is "natural," because the potential in Eq. (5.1) is the most general quartic potential with the G_{exact} symmetry.

Here G_{exact} acts transitively on $G_{\text{approx}}/H_{\text{approx}}$, so the alignment problem is trivial. Any Higgs field in the $G_{\text{approx}}/H_{\text{approx}}$ can be rotated by a G_{exact} transformation to the standard form

$$\phi_0=\frac{1}{\sqrt{2}}\begin{bmatrix}v\\ 0\end{bmatrix}\ . \tag{5.10}$$

The gauge symmetry is completely broken, and the unbroken exact symmetry is

$$H_{\text{exact}}=\mathrm{U}(1)_V\rtimes Z_{2,V}\ , \tag{5.11}$$

where $\mathrm{U}(1)_V$ is generated by

$$Q=Y_L+Y_R \tag{5.12}$$

and the $Z_{2,V}$ is generated by the charge-conjugation operation

$$\mathcal{C}_V:\phi\to\phi^* \tag{5.13}$$

that anticommutes with Q. Of the three Nambu-Goldstone bosons, one is absorbed, and the other two remain exactly massless. The $\mathrm{U}(1)_R$ vector boson acquires the mass

$$\mu^2=\tfrac{1}{4}e^2v^2\ . \tag{5.14}$$

This model has no stable domain walls or monopoles, but it has a topologically conserved magnetic flux classified by $\pi_1(\mathrm{U}(1)_R)=Z$. The vortex configuration with unit flux has the asymptotic behavior

$$\phi(r=\infty,\theta)=\frac{v}{\sqrt{2}}\begin{bmatrix}e^{i\theta}\\ 0\end{bmatrix}\ . \tag{5.15}$$

Since $\pi_1(G_{\text{exact}}/H_{\text{exact}})=0$, the first homomorphism in Eq. (3.2) has trivial kernel, and a cylindrically symmetric "Skyrmion" configuration can be constructed that has this asymptotic behavior, and lives in the exact vacuum manifold everywhere; it is

$$\begin{aligned}\phi^{(\text{Skyrmion})}(r,\theta)&=\frac{v}{\sqrt{2}}(r^2+a^2)^{-1/2}\begin{bmatrix}re^{i\theta}\\ a\end{bmatrix}\ ,\\ \frac{e}{2}A_\theta^{(\text{Skyrmion})}(r)&=\frac{-r^2}{r^2+a^2}\ ,\end{aligned} \tag{5.16}$$

where a is an arbitrary distance scale, and e is the gauge coupling. As Hindmarsh [16] observes (see also Ref. [14]), the exact vacuum manifold, with the gauged $\mathrm{U}(1)_R$ moded out, is the manifold $\mathrm{CP}^1=S^2$, and Eq. (5.16) is the Skyrmion solution of the CP^1 σ model in two spatial dimensions. Its covariant gradient energy (in two dimensions) is

$$m_{\text{Skyrmion}}=\pi v^2\ , \tag{5.17}$$

which is thus the energy of the configuration with the magnetic flux spread out to infinity.

There is also a Nielsen-Olesen [17] vortex solution, with $\phi=0$ at the origin. Its mass equals the Skyrmion mass for $\beta\equiv\lambda/(e/2)^2=m_S^2/\mu^2=1$, and it is lighter than the Skyrmion for $\beta<1$ [18]. Thus, there is a stable vortex for $\beta<1$. But for $\beta>1$, the Nielsen-Olesen vortex solution is heavier than the Skyrmion, and the vortex is unstable. The analysis of Hindmarsh [16] and of Achúcarro *et al.* [19] indicates that there are no metastable vortices in this model. For $\beta>1$, the vortex is classically unstable, and the magnetic flux wants to spread out.

B. Quantum stability

Now consider gauging

$$G_{\text{gauge}}=[\mathrm{U}(1)_L\times\mathrm{U}(1)_R]/Z_2 \tag{5.18}$$

generated by Y_L and Y_R. The exact symmetry of this model is

$$G_{\rm exact}=([\mathrm{U}(1)_L \otimes Z_{2,L}]\times[\mathrm{U}(1)_R \otimes Z_{2,R}])/Z_2 \ , \tag{5.19}$$

where the Z_2's are generated by the charge-conjugation operations

$$\begin{aligned} \mathcal{C}_L &: \phi \to -i\sigma_2\phi \ , \\ \mathcal{C}_R &: \phi \to -i\sigma_2\phi^* \ ; \end{aligned} \tag{5.20}$$

$\mathcal{C}_L$ flips the sign of Y_L, and $\mathcal{C}_R$ flips the sign of Y_R.

Unfortunately, the $G_{\rm approx}$ symmetry is not natural in this model. The quartic interaction term $(|\phi_1|^2-|\phi_2|^2)^2$ is invariant under $G_{\rm exact}$, but has not been included in Eq. (5.1). I will nevertheless analyze the effect of symmetry-breaking quantum corrections in this model, to illustrate the earlier general discussion. Natural models can be constructed (notably including models without elementary scalars, in which the spontaneous breakdown of $G_{\rm approx}$ is *dynamical),* but they are more complicated to construct and analyze. Examples will be discussed in Sec. VI.

There is a nontrivial alignment problem in this model, which we can resolve by minimizing the one-loop effective potential. If the Higgs doublet has the vacuum expectation value

$$\langle\phi\rangle\equiv\phi_0=\frac{1}{\sqrt{2}}\begin{bmatrix} v_1 \\ v_2\end{bmatrix}, \quad |v_1|^2+|v_2|^2=v^2 \ , \tag{5.21}$$

then the tree-level gauge-boson mass matrix is

$$\mu^2=\frac{1}{4}\begin{bmatrix} e_L^2(|v_1|^2+|v_2|^2) & e_Le_R(-|v_1|^2+|v_2|^2) \\ e_Le_R(-|v_1|^2+|v_2|^2) & e_R^2(|v_1|^2+|v_2|^2)\end{bmatrix} , \tag{5.22}$$

where $e_{L,R}$ are the gauge couplings, and the leading (in $\hbar$) term in the effective potential that depends on the alignment is [20]

$$V_{\rm eff}=\frac{3}{64\pi^2}\mathrm{Tr}\mu^4\ln(\mu^2/M^2) \ . \tag{5.23}$$

where M is a renormalization scale.

A symptom of the unnaturalness of this model is that the statement that the $G_{\rm approx}$ symmetry is a good symmetry at the classical level is really dependent on the choice of the scale M, for shifting the renormalization scale moves symmetry-breaking terms in the potential from the one-loop term to the tree term. We suppose that the classical potential is $G_{\rm approx}$ invariant when M is of order the symmetry-breaking scale v. Then the minimum of the potential occurs for $|v_1|^2|v_2|^2=0$, if the gauge couplings are weak. By a $G_{\rm exact}$ transformation, we can therefore choose ϕ_0 as in Eq. (5.10). Thus the unbroken symmetries are

$$H_{\rm gauge}=\mathrm{U}(1)_V, \quad H_{\rm exact}=\mathrm{U}(1)_V\otimes Z_{2,V} \ , \tag{5.24}$$

with the $\mathrm{U}(1)_V$ and $Z_{2,V}$ generators defined as in Eqs. (5.12) and (5.13). The vector-boson spectrum is

$$\begin{aligned} \mu_Z^2 &=\tfrac{1}{4}(e_L^2+e_R^2)v^2, \quad Z=B_L\cos\theta-B_R\sin\theta \ , \\ \mu_A^2 &=0, \quad A=B_L\sin\theta+B_R\cos\theta \ , \end{aligned} \tag{5.25}$$

where B_L and B_R are the $\mathrm{U}(1)_L\times\mathrm{U}(1)_R$ gauge bosons, and θ is the mixing angle defined by $\tan\theta=e_R/e_L$. Of the three Nambu-Goldstone bosons, one is absorbed, and the other two become a charged pseudo Goldstone boson with mass

$$m_{\rm PGB}^2=-\frac{3}{128\pi^2}e_L^2e_R^2v^2\ln[(e_L^2+e_R^2)v^2/4M^2] \ ; \tag{5.26}$$

the mass goes to zero as either gauge coupling turns off.

Since there is a spontaneous broken U(1) gauge symmetry, there is a topologically conserved magnetic flux, and vortex configurations can be constructed that have Z flux trapped in the core. If we ignore the quantum corrections to the effective potential, the stability of the vortices can be analyzed just as above, except that the critical coupling becomes $\beta=\lambda/[(e_L^2+e_R^2)/4]=m_S^2/\mu_Z^2=1$. For $\beta>1$, the vortex becomes classically unstable, and the magnetic flux wants to spread. But in this model, the one-loop corrections prevent the flux from spreading to infinity. The vortex, stabilized by quantum corrections, has a core size

$$r_{\rm core}\sim m_{\rm PGB}^{-1} \ . \tag{5.27}$$

This model also contains domain walls, because the exact vacuum manifold has two disconnected components—one with $|v_1|^2=0$ and one with $|v_2|^2=0$. Ignoring quantum corrections, the domain wall is unstable; it can lower its gradient energy by spreading out. But the one-loop corrections prevent it from spreading beyond a size given by Eq. (5.27).

C. Semilocal monopole

Now suppose that the gauge group is

$$G_{\rm gauge}=\mathrm{SO}(3) \ , \tag{5.28}$$

under which $\pi^a, a=1,2,3$, is a triplet and π^0 is a singlet. Then the exact symmetry is

$$G_{\rm exact}=\mathrm{O}(3) \tag{5.29}$$

[which includes a parity transformation, the element, $\mathbb{1}$ in SO(4)].

Again, this model is unnatural; we can add any even function of π^0 to the potential in Eq. (5.1) without breaking the $G_{\rm exact}$ symmetry. Still, we may proceed as in Sec. V B, imposing the symmetry at a renormalization scale of order v, and solving for the alignment by minimizing the one-loop effective potential. We then find that the minimum occurs for $\pi^0=0$, so that the unbroken gauge symmetry is

$$H_{\rm gauge}=\mathrm{SO}(2) \ . \tag{5.30}$$

(For a caveat concerning this alignment, see the discussion of dynamical symmetry breaking in Sec. VI.) Thus, $\pi_2(G_{\rm gauge}/H_{\rm gauge})=Z$, and this model contains magnetic monopoles.

Since $\pi_2(G_{\rm approx}/H_{\rm approx})=0$, the homomorphism Eq. (4.3) has a trivial kernel. This means that there are magnetically charged configurations such that the Higgs field takes values in the approximate vacuum manifold everywhere. A spherically symmetric configuration of this type that carries one unit of magnetic charge is

$$\pi^0=v\sqrt{1-f(r)^2}\ ,\qquad \pi^a=vf(r)\hat{r}^a,\quad a=1,2,3\ , \tag{5.31}$$

where

$$f(\infty)=1,\quad f(0)=0\ . \tag{5.32}$$

This configuration can lower its gradient energy by shrinking, but it is prevented from collapsing completely by its magnetic Coulomb energy. If π is constrained to take the form Eq. (5.31), then the energy will be minimized when the size is of order $(ev)^{-1}$ (where e is the gauge coupling), and the mass of the monopole is of order $4\pi v/e$.

There are also "'t Hooft–Polyakov" configurations [21]:

$$\pi^0=0,\quad \pi^a=vg(r)\hat{r}^a,\quad a=1,2,3\ , \tag{5.33}$$

where

$$g(\infty)=1,\quad g(0)=0\ . \tag{5.34}$$

Such a configuration has Higgs field potential energy in its core, and the energy is minimized by the usual 't Hooft–Polyakov solution.

In the Bogomol'nyi limit $\lambda/e^2\to 0$ [18,22], the Higgs potential energy is negligible, and the monopole of minimal energy has the form Eq. (5.33). Turning on π^0 only increases the gradient energy. But in the opposite limit $\lambda/e^2\to\infty$, the form Eq. (5.31) has lower energy. To see this, note that in the limit of large λ, the Higgs field core of the 't Hooft–Polyakov solution shrinks to zero size [23], so that $g(r)=1$, for all r. This solution is then of the form Eq. (5.31), but with $f(r)$ constrained to be 1. One anticipates that, by relaxing this constraint, a lower energy configuration of the form Eq. (5.31) can be found. Indeed, a simple stability analysis shows that the 't Hooft–Polyakov solution becomes unstable for large λ/e^2, and that a π^0 condensate is favored inside the core. Thus, for large λ, the Higgs field inside the monopole core remains close to the approximate vacuum manifold, and the approximate SO(4) symmetry is not "restored" anywhere inside the core. This is a semilocal monopole.

A *natural* model with a semilocal monopole can be constructed as follows: Consider the symmetry-breaking pattern $G_{\rm approx}={\rm SO}(8)\to H_{\rm approx}={\rm SO}(7)$, driven by a Higgs field in the vector representation of SO(8). Now gauge $G_{\rm gauge}={\rm SU}(3)$, embedded so that the Higgs field transforms as the adjoint representation of SU(3). It is easily verified that the most general quartic Higgs potential that is SU(3) invariant also respects an "accidental" SO(8) symmetry [7]. Depending on the alignment, the unbroken gauge symmetry will be either $[{\rm SU}(2)\times{\rm U}(1)]/Z_2$ or $[{\rm U}(1)\times{\rm U}(1)]/Z_2$. Solving for the alignment by minimizing the one-loop effective potential, one finds $H_{\rm gauge}=[{\rm SU}(2)\times{\rm U}(1)]/Z_2$. Since $\pi_2(G_{\rm gauge}/H_{\rm gauge})=Z$ and $\pi_2(G_{\rm approx}/H_{\rm approx})=0$, this model contains semilocal monopoles. Another example will be described in Sec. VI.

D. Unstable Z_2 vortex

The model in Sec. V A has a spontaneously broken ${\rm U}(1)_{\rm gauge}$, and the topologically conserved magnetic flux takes integer values. The model in this section will demonstrate that it is also possible for unstable vortices to occur when the topologically conserved magnetic flux takes values in Z_2.

The approximate global symmetry is $G_{\rm approx}=[{\rm SU}(3)_L\times{\rm SU}(3)_R]/Z_3$, and the Higgs field transforms as the $(3,\bar{3})$ representation; it can be written as a 3×3 matrix Φ transforming as

$$\Phi\to U_L\Phi U_R^\dagger\ . \tag{5.35}$$

We suppose that the Higgs expectation value can be put in the form

$$\langle\Phi\rangle=v\mathbb{1}\ , \tag{5.36}$$

so that the pattern of symmetry breakdown is

$$G_{\rm approx}=[{\rm SU}(3)_L\times{\rm SU}(3)_R]/Z_3\to H_{\rm approx}={\rm SU}(3)_V/Z_3\ . \tag{5.37}$$

(There are now two independent quartic invariants in the most general Higgs potential, and one cubic invariant, but this pattern occurs for a finite range of parameters.)

Now gauge the subgroup ${\rm SO}(3)\subset{\rm SU}(3)_R$. This model is natural, and the alignment problem is trivial. The exact symmetry breaks as

$$G_{\rm exact}={\rm SU}(3)_L^{\rm global}\times{\rm SO}(3)_R^{\rm gauge}\to H_{\rm exact}={\rm SO}(3)_V^{\rm global}\ ; \tag{5.38}$$

the gauge symmetry is completely broken.

Since $\pi_1[{\rm SO}(3)_R^{\rm gauge}]=Z_2$, this model has a topologically conserved Z_2 magnetic flux. But we also have $\pi_1(G_{\rm exact}/H_{\rm exact})=0$. [The noncontractible loop in ${\rm SO}(3)_R^{\rm gauge}$ can be deformed in $G_{\rm exact}$ to a loop that lies in $H_{\rm exact}$.] Thus, there are configurations with nontrivial Z_2 magnetic flux such that the Higgs field lies in the exact vacuum manifold everywhere. According to the general discussion in Sec. III, then, the vortex will be unstable when the gauge coupling is sufficiently weak.

VI. NATURAL MODELS: DYNAMICAL SYMMETRY BREAKING

In some of the models described above, fine-tuning of bare parameters is required to enforce the condition that the $G_{\rm approx}$ symmetry is a good symmetry to zeroth order in $\hbar$. This kind of fine-tuning can be avoided in a broad class of models that contain no elementary scalar fields. In these models, the spontaneous breakdown of the $G_{\rm approx}$ symmetry is *dynamical,* driven by the condensation of fermion pairs.

Of course, the dynamical symmetry breakdown is actually nonperturbative in $\hbar$, rather than "classical." So we need to change our terminology a bit. In these models, the intrinsic breaking of G_{approx} symmetry turns off as the weak G_{gauge} couplings go to zero. The models are natural in the sense that there are no operators of dimension four or less that are invariant under G_{exact}, other than gauge couplings. The only potential symmetry-breaking terms are bare fermion masses, so we need to ensure that the G_{exact} symmetry is sufficiently restrictive to prevent fermion masses from being generated by the G_{gauge} radiative corrections.

For example, QCD with two massless quark flavors has the chiral symmetry

$$G_{\text{approx}}=[\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_V]/Z_2\ , \tag{6.1}$$

which is dynamically broken to

$$H_{\text{approx}}=[\mathrm{SU}(2)_V\times\mathrm{U}(1)_V]/Z_2\ . \tag{6.2}$$

If we now gauge $G_{\text{gauge}}=[\mathrm{U}(1)_L\times\mathrm{U}(1)_R]/Z_2$, the surviving exact symmetry (in fact, the gauge symmetry) is sufficient to forbid any bare quark masses.

We may proceed to determine the vacuum alignment as in Sec. V B, but in one important respect, the previous analysis needs to be reconsidered. The effective potential that we computed before was of order $e^4\ln(1/e^2)$, where e is the G_{gauge} gauge coupling. But there may also be terms in the potential that are of order e^2, and so are the dominant terms at weak gauge coupling. (We did not consider such terms before, because they are not generated until two-loop order in models with elementary scalars.) Fortunately, it is easy to show that no order-e^2 terms arise in the type of model considered here, where no weak gauge bosons couple to both left-handed and right-handed quarks [24]. Thus, our previous analysis of the vacuum alignment is applicable. The new feature is that the effective potential is actually finite, because there are no possible symmetry-breaking counterterms; it has the form Eq. (5.23), where M is the scale of dynamical symmetry breakdown. We conclude that the model contains a vortex and domain wall with thickness given by Eq. (5.27).

The situation is different for our model with a semilocal monopole, in which $G_{\text{gauge}}=\mathrm{SU}(2)_V$. Here the exact symmetry is

$$G_{\text{exact}}=[\mathrm{SU}(2)_V\times\mathrm{U}(1)_V\times Z_{4,A}]/Z_2\ , \tag{6.3}$$

and the axial $Z_{4,A}$ symmetry is sufficient to forbid bare quark masses. But since the weak gauge symmetry is now vectorlike, there is an order-e^2 term in the effective potential. The minimum of this potential occurs when G_{gauge} is unbroken [24,25], contrary to our previous findings, and the model contains no magnetic monopoles.

It is not difficult to construct slightly more elaborate models in which natural semilocal monopoles can occur. For example, the symmetry-breakdown pattern

$$G_{\text{approx}}=\mathrm{SU}(4)\rightarrow H_{\text{approx}}=\mathrm{Sp}(4) \tag{6.4}$$

is expected to occur, in a model that contains four massless fermion flavors that transform as a pseudoreal representation of a strongly coupled gauge group. [Because the representation is pseudoreal, a gauge-invariant bilinear fermion condensate must be antisymmetric in flavor indices, and Sp(4) is the maximal symmetry that preserves a condensate in which all fermions acquire masses.] Now, if we gauge $G_{\text{gauge}}=\mathrm{SO}(4)$ [embedded so that the 4 of SU(4) transforms as a 4 of SO(4)], bare fermion masses are forbidden. The condensate transforms as $(3,1)+(1,3)$ under $G_{\text{gauge}}\sim\mathrm{SO}(3)\times\mathrm{SO}(3)$, and we can use the methods of Ref. [24] to find that the vacuum alignment favors the gauge symmetry breakdown pattern

$$G_{\text{gauge}}=\mathrm{SO}(4)\rightarrow H_{\text{gauge}}=[\mathrm{SU}(2)\times\mathrm{U}(1)]/Z_2\ . \tag{6.5}$$

Since $\pi_2(G_{\text{gauge}}/H_{\text{gauge}})=Z$ and $\pi_2(G_{\text{approx}}/H_{\text{approx}})=0$, this model contains a semilocal monopole. Since the discrete parity symmetry that interchanges the two SO(3) factors [which is embedded in SU(4)] is also spontaneously broken, there is a semilocal domain wall in the model, as well.

In models of dynamical symmetry breakdown, then, semilocal defects are topological defects that can be analyzed using an effective Lagrangian that describes physics well below the scale of the symmetry breakdown, as these examples illustrate. The defects have a characteristic size that is larger than the symmetry-breaking scale by a power of the inverse G_{gauge} coupling.

VII. MIXING AND TWISTING

The models that we have been considering have a G_{exact} symmetry with the local structure[6]

$$G_{\text{exact}}\sim G_1\times G_2\ , \tag{7.1}$$

where G_1 is the gauge group and G_2 is a global symmetry group. In Sec. III, we considered the properties of semilocal vortices in models such that G_2 is a nontrivial continuous group. We saw that, under suitable conditions, there can be topological magnetic flux sectors that contain configurations of finite energy in which the flux is spread out over an arbitrarily large area. Such configurations exist if the Higgs field on the circle at $r=\infty$ traces out a noncontractible path in G_1/H_1, and this path can be contracted in $[G_1\times G_2]/H$, in other words, if the vortex is classified by an element of the kernel of the natural homomorphism

$$\pi_1(G_1/H_1)\rightarrow\pi_1([G_1\times G_2]/H)\ . \tag{7.2}$$

In this section, I will discuss this criterion in a bit more detail. Specifically, I will emphasize the (rather obvious) fact that the kernel can be nontrivial only if "mixing" occurs; that is, there must be a generator of H that is a nontrivial linear combination of a G_1 generator and a G_2 generator.

[6]The general compact symmetry group with this local structure is $G_{\text{exact}}=[G_1\times G_2]/G_{\text{discrete}}$, where G_{discrete} is a discrete invariant subgroup of $G_1\times G_2$. But there is really no loss of generality in assuming $G_{\text{exact}}=G_1\times G_2$, if we allow matter fields that represent G_{discrete} trivially.

To see this, let us recall that a closed loop in the coset space G/H may be expressed as

$$\Phi(\theta)=D[g(\theta)]\Phi_0,\quad g(\theta)\in G\ ,$$
$$g(0)=e,\quad g(2\pi)\in H\ ; \tag{7.3}$$

here Φ is an "order parameter" with stability group H, and D is the representation of G according to which Φ transforms. Thus, closed paths in G/H that begin and end at an arbitrarily selected point Φ_0 are parametrized by paths in G (open, in general), that begin at the identity and end at a point in H. The homotopy classes in $\pi_1(G/H)$, then, are equivalent to topological classes of paths in G that begin at the identity and end in H. There are two types of nontrivial classes—ones that end in the identity component of H (which occur only if G is not simply connected), and ones that do not (which occur only if H is not connected).

In the case $G=G_1\times G_2$, a closed path in $[G_1\times G_2]/H$ can be expressed as

$$\Phi(\theta)=D[g_1g_2(\theta)]\Phi_0,\quad g_1(\theta)\in G_1,\quad g_2(\theta)\in G_2\ ,$$
$$g_1g_2(0)=e,\quad g_1g_2(2\pi)\in H\ , \tag{7.4}$$

where D is the representation of $G_1\times G_2$ according to which Φ transforms. Now, consider a nontrivial element of the kernel of the homomorphism Eq. (7.2). Representing it is a path $g_1^{(0)}(\theta)\in G_1$ that cannot be smoothly deformed so that it lies in H_1 for all θ, if we fix $g_1(0)=e_1$, and require that $g_1(2\pi)\in H_1$. By assumption, it is possible to deform this path so that $g_1g_2(\theta)$ lies entirely in H.

Let us denote this deformation by $g_1g_2(t,\theta)$, where $t\in[0,1]$, and

$$g_1(0,\theta)=g_1^{(0)}(\theta),\quad g_2(0,\theta)=e_2\ ,$$
$$g_1g_2(1,\theta)\in H\ . \tag{7.5}$$

Now we distinguish two cases. If $g_1(t,2\pi)\in H_1$ for all t, then we know that $g_1(1,\theta)$ cannot lie in H_1 [for otherwise $g_1^{(0)}(\theta)$ defines a trivial closed path in G_1/H_1, contrary to our assumption]. But $g_1(1,\theta)g_2(1,\theta)$ *is* in H. So, as θ varies, $D[g_1(1,\theta)]$ and $D[g_2(1,\theta)]$ both act nontrivially on the order parameter Φ_0, while their product acts trivially. This means that there is a generator of H that is a nontrivial linear combination of broken G_1 and G_2 generators—in other words, there is mixing.

On the other hand, suppose that $g_1(t,2\pi)$ does not stay in H_1 for all t. Then, since $g_1(t,2\pi)g_2(t,2\pi)\in H$, we know that, as t varies, $D[g_1(t,2\pi)]$ and $D[g_2(t,2\pi)]$ act nontrivially on Φ_0, while their product acts trivially. Again, we conclude that there is mixing.

It is useful to restate this conclusion in the language of fiber bundles. We noted in Sec. II that the Nambu-Goldstone bosons associated with the vacuum manifold $[G_1\times G_2]/H$ can be divided into two classes—those that are absorbed by the G_1 gauge fields and the surviving Nambu-Goldstone bosons that remain exactly massless. This division defines, locally, a decomposition of the vacuum manifold into a direct product of two spaces—the G_1 gauge orbit and the space $M_{\rm orbit}$ of gauge orbits. In other words, there is a projection map

$$\pi:[G_1\times G_2]/H\to M_{\rm orbit} \tag{7.6}$$

that takes each point of the vacuum manifold to the gauge orbit on which it lies. This map is a *fibration* of the vacuum manifold, with base space $M_{\rm orbit}$, fiber G_1/H_1 (the gauge orbit), and structure group G_1.

Now, the topologically conserved magnetic flux is classified by the fundamental group of the fiber, the gauge orbit. Configurations with nontrivial magnetic flux can "spread out" if there are noncontractible loops in the fiber that can be contracted in the total space of the bundle—that is, if the homomorphism Eq. (7.2) has a nontrivial kernel.

But suppose that there is no mixing—the unbroken group is $H=H_1\times H_2$, where $H_1\subseteq G_1$ and $H_2\subseteq G_2$. Then we have

$$\frac{G_1\times G_2}{H_1\times H_2}=\frac{G_1}{H_1}\times\frac{G_2}{H_2}\ ; \tag{7.7}$$

the vacuum manifold is *globally* a direct product of the gauge orbit G_1/H_1 and the space $M_{\rm orbit}=G_2/H_2$. Thus, noncontractible loops in a gauge orbit evidently remain noncontractible in the total space of the bundle. Vortices with nontrivial magnetic flux cannot spread.

For a vortex to be able to spread, it is necessary (but not sufficient) for the vacuum bundle to be a nontrivial ("twisted") bundle with base space $M_{\rm orbit}$ and fiber G_1/H_1. For the bundle to be twisted, it is necessary (but not sufficient) for mixing to occur.

Magnetic monopoles are classified by noncontractible two-spheres in the gauge orbit. As noted in Sec. IV B, such a two-sphere always remains noncontractible in the total space of the bundle. A magnetic monopole (with nontrivial topological charge) always has a core.

VIII. (GENERALIZED) ELECTROWEAK VORTICES

As noted above, in a magnetic flux sector classified by a nontrivial element of the kernel of the homomorphism Eq. (7.2), there are configurations of finite energy in which the flux is spread out over an arbitrarily large area. It then becomes a dynamical question whether the energy is minimized in this sector by a spread out configuration or a localized vortex. We argued in Sec. III that the spread out configurations are favored at sufficiently weak gauge coupling, but that stable localized vortices may exist if the gauge coupling is not too weak (or the Higgs-boson mass is not too large).

Following Vachaspati [4], let us consider what would happen to such a stable vortex if we were to gauge the global G_2 symmetry. When G_2 is gauged, the vortex no longer carries a topological conserved magnetic flux, so it is bound to become unstable. But we know that the vortex is stable in the limit $e_2\to 0$, where e_2 is the G_2 gauge coupling. It is reasonable to expect that the classical vortex solution remains classically stable for a finite range of values of e_2, though there are presumably quantum mechanical tunneling processes that allow it to decay. As Vachaspati observes, if we gauge the $SU(2)_L$ global symmetry in the model described in Sec. V A, we obtain the standard electroweak model. This model therefore contains metastable "electroweak strings," (although not for realistic values of the Higgs-boson mass and $\sin^2\theta_W$ [26]).

In this section, I will discuss a few features of the theory of such electroweak vortices.

In general, we consider a model with gauge group $G_1 \times G_2$, spontaneously broken to H. If the G_2 gauge coupling e_2 turns off, the gauge group G_1 breaks to H_1, the intersection of G_1 and H. A (generalized) electroweak vortex is a vortex that carries no topologically conserved flux, but becomes topologically stable in the limit $e_2 \to 0$; thus, it is associated with a nontrivial element of the kernel of Eq. (7.2). As is clear from the discussion in Sec. VII, such an object can exist only if there is gauge boson mixing—there must be a generator of H that is a nontrivial linear combination of a G_1 generator and a G_2 generator.

A. Strings ending on monopoles

Let us denote by $Q_{1,2}$ two generators of $G_{1,2}$ that mix. Suppose that the Higgs field Φ^{q_1,q_2} carries charges $q_{1,2}$, so that

$$Q = \frac{Q_2}{q_2} - \frac{Q_1}{q_1} \tag{8.1}$$

is an unbroken H generator. If $B_{1,2}$ are the gauge fields that couple to $Q_{1,2}$, then

$$A = B_1 \cos\theta + B_2 \sin\theta \tag{8.2}$$

is the massless gauge field that couples to eQ, where e is related to the $G_{1,2}$ gauge couplings by

$$\frac{e}{\sin\theta} = e_2 q_2 , \quad \frac{e}{\cos\theta} = -e_1 q_1 . \tag{8.3}$$

The orthogonal gauge field state is

$$Z = -B_1 \sin\theta + B_2 \cos\theta , \tag{8.4}$$

which couples to

$$\begin{aligned} e_Z Q_Z &= \frac{e}{\cos\theta \sin\theta} \left[\frac{Q_1}{q_1} \sin^2\theta + \frac{Q_2}{q_2} \cos^2\theta \right] \\ &= \frac{e}{\cos\theta \sin\theta} \left[\frac{Q_2}{q_2} - Q \sin^2\theta \right] . \end{aligned} \tag{8.5}$$

The Z need not be a mass eigenstate field; it could be a linear combination of massive gauge bosons with different masses. For example, we might have $Z = X \cos\tilde{\theta} + Y \sin\tilde{\theta}$, where X is a mass eigenstate coupling to $e_X Q_X$ and Y is a mass eigenstate coupling to $e_Y Q_Y$. Then Eq. (8.5) is the combination $e_X Q_X \cos\tilde{\theta} + e_Y Q_Y \sin\tilde{\theta}$ that couples to Z. (Note also that G_1 or G_2 could be a product of several commuting factors, each with an independent gauge coupling. Then Q_1, for example, might be a linear combination of generators, each belonging to a different invariant subalgebra of the G_1 Lie algebra.)

Now consider a vortex that has Z magnetic flux $\Psi^{(Z)}$ confined to its core. This means that, at least in a particular gauge, we have

$$P \exp \left[i \oint_C e_a Q^a B^a_\mu dx^\mu \right] = \exp(i e_Z Q_Z \Psi^{(Z)}) , \tag{8.6}$$

where C is a closed path that encloses the vortex. Here B^a_μ has been summed over the $G_1 \times G_2$ gauge fields, and e_a, Q^a are the corresponding gauge couplings and generators. Since the Higgs field Φ^{q_1,q_2} must be covariantly constant and single-valued outside the core, the Z flux is required to be an integer multiple of the flux quantum

$$\Psi^{(Z)}_0 = \frac{2\pi}{e} \cos\theta \sin\theta = -\frac{2\pi \sin\theta}{e_1 q_1} . \tag{8.7}$$

If a particle is covariantly transported around the minimal vortex, it acquires the Aharonov-Bohm phase

$$\exp \left[2\pi i \left[\frac{Q_2}{q_2} - Q \sin^2\theta \right] \right] . \tag{8.8}$$

For a typical charged particle, and a generic value of the mixing angle θ, this is a nontrivial (in fact, transcendental) phase. But it follows from our assumption that the vortex carries no conserved topological charge that Eq. (8.8) is an element of the identity component of H. In two spatial dimensions, this means that it is possible to smoothly deform the vortex configuration (while the energy remains finite) to a configuration that has only massless H magnetic flux. This configuration can then lower its energy to zero by spreading out indefinitely. Thus, though the vortex may be classically stable, it can decay by tunneling quantum mechanically to the configuration with massless magnetic flux.

Similarly, in three spatial dimensions, there are configurations in which the Z flux tube ends on a finite-mass "magnetic monopole," with A magnetic flux spilling out of the end. One may regard the flux tube as a *visible* Dirac string; then the magnetic flux through a sphere enclosing the monopole may be inferred from Eq. (8.8). Let us define the magnetic charge $g_{\rm mag}$ of the monopole so that $4\pi g_{\rm mag}$ is the total A magnetic flux emanating from the monopole; more precisely, let

$$P \exp \left[i \oint_C e Q A^\mu dx_\mu \right] = \exp(4\pi i e Q g_{\rm mag}) , \tag{8.9}$$

where C is a path that encloses the Dirac string of the monopole. We conclude that

$$g_{\rm mag} = \frac{1}{2e} \sin^2\theta . \tag{8.10}$$

Of course, this flux does not satisfy the Dirac quantization condition, because the string is not invisible. [If $\exp(2\pi i Q_2/q_2)$ is a nontrivial element of the identity component of H, there will be some additional magnetic flux, coupling to another H generator, aside from the A flux given by Eq. (8.10).] Note that, in the case of the standard model, Q defined by Eq. (8.1) is actually twice the conventionally normalized electric charge operator, so we have $g_{\rm mag} = \sin^2\theta / e$, if e is the conventionally normalized electromagnetic gauge coupling. Such Z^0 magnetic flux tubes ending on magnetic monopoles were first discussed by Nambu [27].

A classically stable electroweak string can break in a quantum mechanical tunneling process where a pair of monopoles nucleates spontaneously. The decay of metastable electroweak vortices and flux tubes will be further discussed in Ref. [12].

B. Aharonov-Bohm interactions

We have seen that, for generic values of the mixing angle, particles with nonvanishing Q have nontrivial Aharonov-Bohm interactions with electroweak strings. In principle, the charge Q of a projectile could be measured by scattering the projectile off of a string.

Such measurement processes have attracted much recent interest, particularly in the case where the unbroken gauge group $H_{\rm gauge}$ is disconnected [28]. In that case, there are topologically stable strings associated with the "local discrete symmetry." The Aharonov-Bohm interaction can then probe the "quantum hair" of an object. This quantum hair can be measured at long range, but becomes invisible in the classical limit. In the case of an electroweak string, however, the flux of the string is in the identity component of $H_{\rm gauge}$, and the string is not topologically stable. The charges that can be measured in Aharonov-Bohm scattering off the string are not varieties of quantum hair. To be specific, consider the standard model, in which $\exp(2\pi i Q_2/q_2)=1$. Then the Aharonov-Bohm phase (8.8) is completely determined by the charge Q. Therefore, we cannot learn anything about a particle in an Aharonov-Bohm scattering experiment that we could not discern by measuring its classical electric field.

This observation is easily generalized. The effect of transport around a vortex is always described by an element of the unbroken gauge group $H_{\rm gauge}$, because the Higgs condensate must be covariantly constant and single valued outside the vortex. Thus, the Aharonov-Bohm phase acquired by any projectile is always determined by its transformation properties under $H_{\rm gauge}$. The "classical hair" of the projectile determines its charges in the $H_{\rm gauge}$ Lie algebra. This leaves undetermined only the transformation properties under the "local discrete symmetries" that are not in the identity component of $H_{\rm gauge}$. These additional charges cannot be measured in Aharonov-Bohm scattering if the flux of the string is in the identity component. Thus, quantum hair can be measured only with topologically stable strings.

C. Embedded defects

Vachaspati and Barriola [5] have recently pointed out a more general procedure for constructing static solutions to the classical field equations that are not topologically stable. Consider a gauge theory with gauge group $G_{\rm gauge}$ spontaneously broken to the subgroup $H_{\rm gauge}$. Now choose a nontrivial subgroup $\tilde{G}_{\rm gauge}$, such that the intersection of $\tilde{G}_{\rm gauge}$ and $H_{\rm gauge}$ is $\tilde{H}_{\rm gauge}$. Suppose that the natural homomorphism

$$\pi_n(\tilde{G}_{\rm gauge}/\tilde{H}_{\rm gauge})\to\pi_n(G_{\rm gauge}/H_{\rm gauge}) \tag{8.11}$$

has a nontrivial kernel. In other words, there are noncontractible loops ($n=1$) or two-spheres ($n=2$) in $\tilde{G}_{\rm gauge}/\tilde{H}_{\rm gauge}$ that are contractible in $G_{\rm gauge}/H_{\rm gauge}$. Vachaspati and Barriola then show that, under suitable conditions, there are classical vortex ($n=1$) or monopole ($n=2$) solutions to the field equations associated with the nontrivial elements of the kernel. That is, if a gauge theory with gauge group $\tilde{G}_{\rm gauge}$ broken to $\tilde{H}_{\rm gauge}$ contains a topologically stable defect, this defect remains a solution to the field equations when the gauge group is enlarged to $G_{\rm gauge}\supset\tilde{G}_{\rm gauge}$. The electroweak vortices described above are a special case of such "embedded defects," where $G_{\rm gauge}=G_1\times G_2$ and $\tilde{G}_{\rm gauge}=G_1$.

But there is no particular reason, in general, to expect an embedded defect to be classically stable. In the case of embedded monopoles, one can make a stronger statement: they are always classically *unstable* (if not topologically stable).

To see this, we recall that, in a model with unbroken $H_{\rm gauge}$ symmetry, we may associate with any magnetic monopole a topological $H_{\rm gauge}$ charge. The matching condition (or Dirac string) of the monopole defines a closed path in $H_{\rm gauge}$, and the corresponding element of $\pi_1(H_{\rm gauge})$ is the magnetic charge [13,1]. If the monopole arises in a model with an underlying $G_{\rm gauge}$ symmetry, and is nonsingular, then this loop in $H_{\rm gauge}$ must be contractible in $G_{\rm gauge}$. (Otherwise, the Dirac string would necessarily end on a point singularity.) This means that a nonsingular monopole with nontrivial $H_{\rm gauge}$ is always associated with a nontrivial element of $\pi_2(G_{\rm gauge}/H_{\rm gauge})$; it is topologically stable.

Conversely, a monopole that is *not* topologically stable must carry trivial $H_{\rm gauge}$ charge. It was shown by Brandt and Neri [29] and Coleman [30] that such monopoles are always classically unstable. To demonstrate the instability, it suffices to study the small vibrations of the long range $H_{\rm gauge}$ gauge field; it is not necessary to consider the structure of the monopole core. But since there is no topological conservation law to prevent it, the core will presumably "unwind," and its energy will be carried to spatial infinity as non-Abelian radiation.

An embedded monopole is just a particular type of monopole solution that carries no topological charge, and it is therefore unstable.

IX. CONCLUDING REMARKS

A. Semilocality

I have used the term "semilocal" to characterize defects that occur in models in which the gauge group is embedded in a larger group of (approximate) global symmetries. These defects carry "topologically conserved" charges, yet can be deformed so that the order parameter takes values in the approximate vacuum manifold everywhere. This usage encompasses the vortices originally considered by Vachaspati and Achúcarro [3]. It also includes a broader class of domain walls, vortices, and monopoles. These share the feature that the spontaneously broken approximate global symmetry is not restored inside the core of the defect. Indeed, the structure of the defect can be well described using an effective field theory, in which the physics responsible for the spontaneous symmetry breakdown has been "integrated out."

But the term "semilocal" could be and has been used in other ways. Hindmarsh [31] defines a semilocal defect as one that arises in a model such that the vacuum manifold is a twisted bundle of gauge orbits, as described in Sec. VII. This classification leads him to consider an interesting "semilocal texture" contained in the model of Ref. [3] and Sec. V A. I have not discussed textures in this paper, as the emphasis has been on defects that carry topologically conserved charges. (Textures, in contrast, even if classically stable, can "unwind" via quantum tunneling.)

B. Semilocal strings

Although I have broadened the notion of a semilocal defect here, it should be noted that the most interesting kind of semilocal defect is still the semilocal vortex analyzed in Ref. [3]. I would like to emphasize what was truly surprising and noteworthy (to me) about Ref. [3]. It was not, I think, that a stable vortex could exist even though the vacuum manifold is simply connected. It had been stressed by Coleman,[7] and was widely appreciated, that only the pattern of *gauge* symmetry breaking is relevant to the classification of finite-energy vortices. The surprise was not that a semilocal vortex could be *stable,* but that it could be *unstable.* Part of the motivation for this work came from the desire to understand better why the magnetic flux wants to spread out when the gauge coupling is sufficiently weak. (It is also nicely explained in Hindmarsh's papers [16,31].)

C. Electroweak strings

Having said that the surprising feature of semilocal vortices is that they can decay, I should admit that the implications of the existence of *stable* semilocal vortices are quite interesting. As Vachaspati [4] emphasized, a stable semilocal vortex will remain *classically* stable even if the global symmetry is gauged, provided the gauge coupling is not too large. Unfortunately, classically stable strings do not arise in the minimal standard model, for realistic values of the $\sin^2\theta_W$ and Higgs-boson mass [26]. But they may well occur in realistic extensions of the standard model. Thus, we are invited to contemplate the consequences of long-lived metastable strings at the electroweak scale.

First, there would be new resonances at the TeV scale. These could be segments of string with monopoles at the ends (as envisioned long ago by Nambu [27]), or closed loops of string. Regrettably, since these states are "squishy" classical objects, production of the new resonances would be highly suppressed in hard pointlike collisions. They are not likely to be seen in future accelerator experiments.

Second, the strings would be produced during the electroweak phase transition in the early Universe. Not many strings would survive to the present epoch, though. Because the strings can end on monopoles, the strings that are initially produced in the phase transition will be predominantly short open segments and small closed loops [32]. Crudely speaking, each string has a nonzero probability per unit length of ending (on a monopole), so that long strings are exponentially suppressed. The string-monopole network is therefore expected to disappear quickly. The main cosmological implications of the strings, then, would concern their influence on the electroweak phase transition itself, perhaps including their impact on electroweak baryogenesis [33].

[7]See Appendix 3 of Ref. [1].

D. Electroweak flux tubes and the monopole problem

Another potential cosmological implication of electroweak strings deserves comment. Lazarides and Shafi [34] suggested many years ago that electroweak flux tubes might offer a natural solution to the cosmological monopole problem [35]. The idea is that the grand unified theory (GUT) monopoles that are copiously produced in the very early Universe might become confined by flux tubes after the electroweak phase transition. The flux tubes would greatly enhance the rate of monopole annihilation, and rapidly reduce the monopole abundance to an acceptable level.

There are some problems with this idea. First, as Lazarides and Shafi noted [34], the magnetic monopoles in the simplest grand unified models carry $U(1)_{\text{electromagnetic}}$ magnetic charge and $SU(3)_{\text{color}}$ magnetic charge. They do not have any Z^0 magnetic flux, and they are little affected by the electroweak phase transition. Still, there are alternative models in which the stable magnetic monopoles carry $U(1)_{\text{hypercharge}}$ magnetic charge (as well as color magnetic charge). These monopoles have both Z^0 and A magnetic flux, so that the Lazarides-Shafi mechanism might work.

A second problem is that the Z^0 flux tubes are unstable in the simplest models, so that monopole confinement does not really occur, even if the monopoles do have Z^0 magnetic fields. But we have noted that the Z^0 flux tubes could be stable in extended models, so it still seems that there is a class of models in which the Lazarides-Shafi mechanics could work.

There is a third problem however, that probably makes the idea untenable, even under optimistic assumptions. The problem is that an electroweak flux tube can end on either a heavy GUT monopole or on a light electroweak (Nambu) monopole. There is no guarantee, then, that the flux tube emanating from a GUT monopole will bind it to another GUT monopole, rather than to a light electroweak monopole.

The GUT monopole with minimal $U(1)_{\text{hypercharge}}$ magnetic charge carries electromagnetic charge $\cos^2\theta/e$, in addition to its confined Z^0 flux. If the flux tube ends on an electroweak monopole with charge $\sin^2\theta/e$, then the monopole-string composite has magnetic charge $1/e$, twice the Dirac charge. After the flux tube shrinks away, this object becomes an unconfined stable magnetic monopole, with electromagnetic (and color) magnetic charge.

For the Lazarides-Shafi mechanism to successfully reduce the monopole abundance to an acceptable level, electroweak monopoles must be heavily suppressed, so that the flux tubes almost always end on GUT monopoles. It seems difficult to devise a plausible scenario of this kind.

As this paper was being completed, I became aware of Ref. [31], which has some overlap with the research reported here.

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