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Quantum Hair*

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Abstract

A black hole can carry "quantum hair" that, while undetectable classically, can influence quantum physics outside the event horizon. The effect of quantum hair on black hole thermodynamics is computed here, in a particular field-theoretic model.

1. Introduction

The remarkable black hole uniqueness theorems† assert that a stationary black hole can be completely characterized by its mass, angular momentum, and electric charge. Since a black hole carries (almost) no "hair," an observer outside the event horizon is (almost) completely ignorant of the black hole's internal state. This absence of hair has therefore been cited as the explanation of the intrinsic black hole entropy that was conjectured by Bekenstein [2] and calculated by Hawking [3].

However, it is important to remember that the "no-hair theorems" are statements about the solutions to *classical* field equations. These theorems do not exclude the possibility that black holes can carry additional attributes that influence quantum physics outside the horizon, but cannot be described in classical language. Such "quantum hair," were it to exist, would require us to re-examine our ideas about black hole thermodynamics, and about quantum mechanics in a black hole background.

In this talk, I will describe a possible variety of quantum hair that has recently attracted attention, and will explore some implications concerning black hole physics.

The classical no-hair theorems invite us to propose a more general no-hair *principle* – that when an event horizon forms, any feature of a black hole that *can* be radiated away *will* be radiated away [4]. Properties that *cannot* be radiated away are those that can be detected, at least in principle, at infinite range. With this motivation, the term "hair" might be used, in a broader context than black hole physics, to mean an attribute of a localized object that can be measured at arbitrarily long range. Hair, in this sense, can be either classical or quantum-mechanical. Classical hair, like electric charge, is associated with a long-range classical field. Quantum hair is associated with a long-range Aharonov–Bohm phenomenon that has no classical analog. The quantum hair that will be considered here is of this Aharonov–Bohm type.

Of course, it is quite possible that, in order to give a complete description of quantum mechanics on a black hole background, it will be necessary to ascribe to the black hole many additional properties that have nothing to do with the Aharonov–Bohm effect. In other words, a black hole might have many varieties of quantum hair that are outside the scope of the analysis in this talk. This analysis is not intended to be exhaustive, but it will demonstrate that quantum hair can exist, and that it can have dramatic and computable effects on the thermodynamic behaviour of a black hole. We will also gain some insight into the issue of the final state of an evaporating black hole.

2. Quantum hair and the Aharonov–Bohm effect

Krauss and Wilczek [5] pointed out that quantum hair may arise in a gauge theory as a consequence of the Higgs mechanism. To illustrate this phenomenon, we consider a theory with gauge group U(1) that contains both a charge-N scalar field η and a charge-1 scalar field ϕ; under a gauge transformation parametrized by $\omega(x)$, these fields transform as

$$\omega\colon \phi \to e^{i\omega}\phi, \quad \eta \to e^{iN\omega}\eta. \tag{2.1}$$

Suppose that (in unitary gauge) η condenses in the vacuum state, and ϕ does not,

$$\langle\eta\rangle = v \neq 0, \quad \langle\phi\rangle = 0. \tag{2.2}$$

Then the Higgs mechanism occurs; the photon acquires a mass $\mu \sim Nev$ and the classical electric field of a charge decays at long range like

$$E \sim e^{-\mu R}, \quad \mu R \gg 1. \tag{2.3}$$

However, the local U(1) symmetry is not completely broken. There is a surviving manifest Z_N local symmetry under which the scalar fields transform according to

$$\eta \to \eta, \quad \phi \to e^{2\pi i k/N}\phi, \quad k = 0, 1, 2, \ldots, N-1. \tag{2.4}$$

The physical content of the surviving Z_N symmetry is that, because the condensate that screens the classical electric field of a charge is a condensate of charge-N η particles, the condensate is not capable of screening the charge modulo N.

But if the classical electric field of any charge decays according to eq. (2.3), what does it mean to say that the charge modulo N is unscreened? That is, how can the charge modulo N be detected at long range? The key point is that the Higgs phase with manifest Z_N local symmetry supports a "cosmic string." The core of this string traps a quantity of magnetic flux Φ_0/N, where $\Phi_0 = 2\pi/e$ is the flux quantum associated with a particle of charge 1. (Hence e is the gauge coupling.) Thus, when a charge-1 ϕ particle winds around the string, it acquires the nontrivial Aharonov–Bohm phase [6]

$$\exp\left(ie \oint_C A \cdot dx\right) = e^{2\pi i/N}. \tag{2.5}$$

This Aharonov–Bohm phase induces a long-range interaction between the string and the ϕ particle. For example, although the string has a thickness of order μ^{-1}, a ϕ particle incident on the string with energy $E \ll \mu$ scatters with a

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† See [1] for references.

cross-section that is independent of the string thickness [6, 7]. Similarly, a closed loop of string that winds around an object with nonzero Z_N charge acquires a nontrivial Aharonov–Bohm phase, and so the charge scatters an incident string loop.

Even though the photon has acquired a mass due to the Higgs mechanism, and classical electric fields are screened, the Aharonov–Bohm interaction is of infinite range. This interaction enables us, in principle, to measure the Z_N charge of an object while maintaining an arbitrarily large separation from the object. Thus, Z_N charge is a type of quantum hair [5].

That such infinite-range phenomena can occur in the Higgs phase, even though there is a mass gap, has a number of interesting consequences. For one, the existence (or not) of a long-range Aharonov–Bohm interaction provides a criterion for distinguishing different phases of a gauge theory – in the model just described, we have seen that two different Higgs phases, with manifest and spontaneously broken Z_N symmetry, can be distinguished in this manner. This observation is readily generalized to other models [8].

Furthermore, fascinating new phenomena occur in models such that the unbroken gauge symmetry in the Higgs phase is nonabelian. There can be infinite-range processes in which charges are transferred between point particles and string loops, by means of the nonabelian Aharonov–Bohm effect [8, 9].

But our main interest here is in the implications of Z_N quantum hair for black hole physics, to which we now turn.

3. The no-hair theorem and the Higgs mechanism

In a U(1) gauge theory in the Coulomb phase (in which the photon is massless), an electrically charged particle has an infinite-range electric field. Gauss's law ensures that his long-range electric field cannot be extinguished when the particle disappears behind the event horizon of a black hole. Thus we learn that a black hole must be able to carry electric charge; electric charge is a type of classical hair.

Now suppose that the Higgs mechanism occurs, so that the photon acquires a mass μ. The electric field of a charge decays at long range like $E \sim e^{-\mu R}$, where R is the distance from the charge. But if a charged particle drops into a black hole, the event horizon refuses to support a static electric field, and the exponentially decaying long-range field cannot survive [10]. The no-hair principle ("whatever can be radiated *is* radiated") requires the electric flux to leak away from the horizon, either by propagating through the horizon or by escaping to infinity. If the photon Compton wavelength μ^{-1} is much larger than the black hole radius R_{BH}, then the characteristic time scale (in terms of Schwarzschild time t) for this leakage process is of order μ^{-1}.* The analysis leading to the above conclusion is purely classical, and it is not affected if the Higgs mechanism leaves unbroken a local Z_N symmetry. Once the photon acquires a mass, regardless of any surviving manifest discrete symmetry, a stationary black hole that is nonsingular at the event horizon must have a *vanishing* classical electric field outside the horizon.

But this classical analysis cannot exclude the possibility that a black hole may carry quantum hair. Indeed, we know that a particle with nonzero Z_N charge (electric charge modulo N) has an infinite-range Aharonov–Bohm interaction with a cosmic string. This long-range interaction cannot be extinguished when the particle disappears behind the event horizon of a black hole. Thus we learn that a black hole must be able to carry Z_N charge;† Z_N charge is a type of quantum hair.

It is essential to recognize that the Aharonov–Bohm interaction remains undiminished even as the classical electric field decays. This is already evident when we note that the phase acquired by a loop of string that winds around a Z_N charge is nontrivial when the trajectory of the string stays arbitrarily far from the charge, even though the electric field encountered by the string is exponentially small. Likewise, the phase acquired by a loop of string a fixed distance from a charge persists even after the charge crosses the black hole horizon, and the electric field leaks away.

Aside from the Z_N quantum hair described here, another exotic type of black hole hair has attracted attention recently – axionic hair [11]. Rather than digress here on this topic, I have relegated a comparison of Z_N hair and axionic hair to an appendix.

4. Challenges posed by black hole radiance

Having established that a black hole can carry a conserved Z_N charge, even though no massless gauge field couples to the charge, we now wish to consider how the Z_N charge influences the Hawking radiation emitted by the black hole. Our hope is that some of the mysterious aspects of black hole radiance can be illuminated by this investigation.

Specifically, the semiclassical theory of black hole radiance developed by Hawking [3] raises (at least) three fundamental issues, none of which can be satisfactorily addressed within the context of the leading semiclassical approximation. These issues, enumerated below, are all closely related but are logically distinct.

4.1. *The loss of quantum coherence*

In Hawking's semiclassical theory, the radiation emitted by an evaporating black hole is described by a density matrix that is exactly thermal [12]. Since the outgoing radiation is in a mixed state, one might argue (as Hawking [13] does) that scattering of quanta off a black hole cannot be described in the usual S-matrix language – an incoming pure state can evolve into an outgoing mixed state. Similarly, a black hole that forms from collapse in a state that is initially pure will eventually evaporate and yield a mixed final state. A black hole, then, appears to provide a means of destroying quantum-mechanical phase information. If so, quantum mechanics as currently formulated is not applicable to processes involving black holes, and must be replaced by some more general formalism.

This remarkable suggestion is much too radical to be accepted lightly; we are obligated to subject it to the closest scrutiny. In particular, the argument for a loss of quantum coherence presumes that the thermal character of the outgoing density matrix computed by Hawking is an intrinsic property of the final state, and not a mere artifact of the approximations that are made in the leading semiclassical

* I thank Kip Thorne for a helpful discussion about this.

† As Krauss and Wilczek [5] noted.

theory. The calculations performed to date do not, I think, exclude the possibility that the outgoing radiation, when examined with sufficient care, actually carries all of the complex phase information contained in the initial state from which the black hole formed. But if quantum coherence is indeed preserved in black hole processes, it seems to be necessary for black holes to be able to carry non-classical varieties of hair.* Only then is it reasonable to claim that the radiation emitted early on leaves an imprint on the black hole that can influence the radiation emitted later, thus establishing quantum-mechanical correlations between radiation emitted at different times.

4.2. *The origin of black hole entropy*

A remarkable consequence of Hawking's semiclassical theory of black hole radiance is that a black hole has an enormous intrinsic entropy (as Bekenstein [2] had anticipated). Efforts to attach a sensible physical interpretation to this entropy have been partially successful,† but I think that the ultimate origin of the black hole entropy remains quite obscure. In general, the statistical-mechanical entropy of a system arises because the system has access to many microscopic internal states as it undergoes thermal fluctuations. Unless black holes are fundamentally different than other systems, then, it should be possible to relate the black hole entropy to a large number of accessible black hole microstates. Because a classical black hole has no hair, the intrinsic entropy suggests that a black hole has many non-classical degrees of freedom.

The black hole entropy, then, like the apparent loss of quantum coherence, hints at the existence of "quantum hair" on black holes. It does not seem likely that all of the entropy

$$S_{BH} = \tfrac{1}{4}A_{BH}, \tag{4.1}$$

where A_{BH} is the area of the event horizon in Planck units, can be attributed to hair of the Aharonov–Bohm type that we will consider here. Rather, eq. (4.1) tempts one to conjecture that the quantum-mechanical internal state of a black hole can be described in terms of a membrane stretched over the horizon that is approximately one Planck length deep and is characterized by about one bit of information per Planck volume. Such a picture has been advocated by 't Hooft [14].

4.3. *The final state*

An evaporating black hole loses mass. Eventually, as the mass approaches the Planck mass M_P, Hawking's semiclassical theory breaks down, for quantum fluctuations in the background geometry can no longer be neglected. At this point, we are unable to predict what will happen without a deeper understanding of quantum gravity than we currently possess.

The black hole may evaporate completely, leaving behind no trace other than the emitted radiation. It is also conceivable that the evaporation halts, leaving behind a stable black hole remnant. This remnant would be a new type of "elementary" particle, presumably with a mass comparable to M_P. Whether a black hole evaporates completely may have important implications for quantum cosmology – Hawking has argued that if a black hole can form and subsequently disappear, then we have strong evidence that fluctuations in the topology of spacetime must be included in quantum gravity [19].

The three issues outlined above are all of fundamental interest. They all also have in common that they cannot be adequately addressed within the leading semiclassical theory of black hole radiance, and so they provide us with powerful motivation for improving on that theory.

5. Black holes with Z_N quantum hair

Our desire for a better grasp of black hole physics beyond the leading semiclassical approximation leads us back to the black hole with Z_N charge described in Section 3. The black hole with Z_N charge provides a model in which dramatic corrections to the leading semiclassical theory are expected. Furthermore, to a limited extent, these corrections can be studied in a systematic and well-controlled approximation.

To see that dramatic corrections should be expected, let us suppose that $N \gg 1$, so that it is possible for the Z_N charge Q to be a large number. Suppose further that the only elementary particles with nonzero Z_N charge have charge 1 and mass m, where m is small compared to the Planck mass M_P.

By assembling Q charge-1 particles, with $N/2 > Q \gg 1$, one can create a black hole with charge Q and mass $M_{BH} \sim Qm$; if N is sufficiently large, we can also arrange that $M_{BH} \gg M_P$, so that semiclassical theory can safely be applied to this black hole. The black hole then loses mass by radiating gravitons, say, but does not radiate away its Z_N charge. Eventually, M_{BH} becomes less than, perhaps much less than, Qm, yet is still comfortably above M_P.

At this point, complete evaporation of the black hole to elementary quanta is no longer possible; it is *kinematically* forbidden. The Z_N charge is exactly conserved, and there is no available decay channel with Z_N charge Q and a sufficiently small mass. Inevitably, then, the evaporation process must stop, leaving behind a stable remnant.‡ We may regard the remnant as an exotic "nucleus," a highly relativistic bound state of Q elementary charged particles.

However, as noted in Section 3, the Z_N quantum hair has no effect whatsoever on the classical geometry of a stationary black hole. The leading semiclassical theory is just free quantum field theory on the background geometry of the black hole, and so it too is unaffected by the Z_N hair. Indeed, the Z_N hair has no effect to any finite order in the semiclassical expansion in $\hbar$. To understand the mechanism by which the Z_N hair inhibits the evaporation of a black hole, we must investigate physics that is non-perturbative in $\hbar$.

6. A remnant stabilized by classical hair

Before we continue the discussion about Z_N quantum hair, it will be enlightening to consider an example of a black hole remnant that is stabilized by *classical* hair – an electrically charged black hole.§ We can concoct a model in which the

* This point has been especially stressed by 't Hooft [14]. See [15–17] for other critiques of the proposal in [13].

† I am thinking in particular of the "thermal atmosphere" picture developed in [18].

‡ It is a logical possibility that the charge-Q object will be able to decay into black holes of lower charge and mass, but the existence of at least one species of exotic stable remnant is guaranteed.

§ In the case where the photon is massless.

complete evaporation of a charged black hole is kinematically forbidden. But when the hair is classical, the mechanism by which evaporation stops *can* be discussed in terms of familiar semiclassical theory.

In the Coulomb phase of electrodynamics, electric charge on a black hole has two effects, both of which can be described using classical language [20]. First, since the charged black hole has an electric field outside the horizon, and the electric field carries stress-energy, the electric charge modifies the black hole geometry. Specifically, if two black holes have the same mass, the one with more charge has a smaller surface gravity κ, and so a smaller Hawking temperature $T_{BH} = \kappa/2\pi$. Second, a charged black hole has a nonzero electrostatic potential at the event horizon; if the black hole is positively charged, say, then more work is required to carry a positive charge than a negative charge from spatial infinity to the horizon. In black hole thermodynamics, the electrostatic potential plays the role of a chemical potential. A positively charged black hole cannot be in equilibrium with a neutral plasma, because it prefers to emit positive charge and to absorb negative charge.

As a charged black hole evaporates, it loses mass. If we suppose that its charge Qe remains fixed as it evaporates,* then the charge to mass ratio Qe/M_{BH} steadily increases. This ratio asymptotically approaches the critical value 1 (in Planck units), at which the surface gravity κ and Hawking temperature T_{BH} vanish.† Heuristically, a black hole of given mass M_{BH} has a maximal charge because, when the Coulomb energy of order $(Qe)^2/R_{BH}$ stored in the electric field outside the horizon becomes comparable to M_{BH}, there is no mass left over at the center to support the horizon. (In fact, as the charge to mass ratio increases toward one, the Cauchy horizon interior to the event horizon asymptotically approaches the event horizon. In this instance, then, the cosmic censorship hypothesis [21] can be identified with the third law of thermodynamics. A third law of black hole dynamics has been proved, under suitable assumptions, by Israel [22]).

We see that the Hawking evaporation process ceases to operate for a maximally charged black hole. Nevertheless, in the real world, a maximally charged black hole would continue to radiate both charge and mass [20]. The reason is that the electrostatic potential energy

$$U = eM_P \tag{6.1}$$

of a positron at the horizon of a positive critically charged black hole is enormous compared to the electron mass. Hence, dielectric breakdown of the vacuum occurs outside the horizon. It is energetically favourable to produce an e^+e^- pair, allowing the electron to be absorbed by the black hole while the positron is ejected to infinity with an ultrarelativistic velocity. The black hole would discharge rapidly.

But since our interest here is in matters of principle, we are free to contemplate a fictitious world with

$$Gm^2 > e^2 \tag{6.2}$$

where m, e are the electron mass and charge. ‡ In this world, the electrostatic interaction between two electrons is weaker than the gravitational interaction. And the electrostatic potential energy of an electron at the horizon of a maximally charged black hole is less than m, so that dielectric breakdown of the vacuum cannot occur. The maximally charged black hole in this model with $M_{BH} = eQM_P$ is lighter than Q electrons, and so its decay to elementary quanta is kinematically forbidden.

The only decay channel potentially available to the maximally charged black hole is a state containing black holes of lower mass and charge. At the classical level, all maximally charged black holes have the same charge to mass ratio, so that such decay modes would be just marginally allowed. However, at least when the black hole radius R_{BH} is much smaller than the electron Compton wavelength m^{-1}, I expect that renormalization effects (particularly charge renormalization) cause the charge to mass ratio of the maximally charged black hole to *increase* with increasing M_{BH}.§ Thus, the particle spectrum of the model should contain a whole tower of stable black hole "solitons" of increasing charge.

It could be quite illuminating to consider in detail the quantum mechanics of scattering processes involving these solitons. On the one hand, it may be feasible, using standard methods of soliton quantization, to construct, order by order in an expansion in $\hbar$, an S-matrix for these processes.¶ On the other hand, if, as Hawking insists, such processes inevitably involve a loss of quantum-mechanical phase information, then no such S-matrix has a right to exist.

7. A world-sheet instanton

In the model just considered, the complete evaporation of a black hole can be kinematically forbidden, if the electric charge on the black hole is large enough, and we were able to understand using semiclassical reasoning why the evaporation of a charged black hole eventually stops. In the model described in Section 5, the complete evaporation of a black hole can be kinematically forbidden, if the Z_N charge on the black hole is large enough. Yet, to any finite order in the expansion in $\hbar$, the Z_N charge has no effect on the evaporation process. Small fluctuations of the quantum fields on the black hole background are completely insensitive to the Z_N charge.

The only readily identifiable quantum fluctuations that *do* depend on the value of the Z_N charge are virtual cosmic strings that wind around the event horizon of the black hole; a string that winds k times around an object with Z_N charge Q acquires the Aharonov–Bohm phase $\exp(2\pi ikQ/N)$. We need a better grasp of the physics of the cloud of virtual strings surrounding a black hole, if we hope to understand the effect of Z_N quantum hair on black hole radiance.

In fact, the effects of interest, which are nonperturbative in $\hbar$, can be incorporated into a systematic small-$\hbar$ approximation. The analysis is done most conveniently using Euclidean path integral methods. And since we are studying quantum field theory on the Schwarzschild background, the

* Loss of charge is negligible if all elementary charged particles are sufficiently heavy; see below.

† We assume, for simplicity, that the angular momentum J_{BH} is zero.

‡ It is conceivable that such an inequality applies, even in the real world, to the mass and charge of a *magnetic monopole*.

§ Loosely speaking, it is the "bare" charge of the black hole, as measured at the distance scale R_{BH}, that enters into the relation $eQ = M$ satisfied by a black hole with $T_{BH} = 0$. The renormalized charge, as measured far away, is partially screened by vacuum polarization, and this charge screening becomes less effective as R_{BH} increases.

¶ This point was emphasized to be by David Gross.

appropriate arena is the Euclidean section of the Schwarzschild geometry [23].

The Euclidean section of the Schwarzschild geometry has the topology of $R^2 \times S^2$. The plane R^2 is parametrized by the Schwarzchild radial coordinate r and the Euclidean Schwarzschild time coordinate τ. These turn out to be polar coordinates for the plane; τ is actually an angular variable, periodic with period β, where $\beta^{-1} = T_{BH}$ is the Hawking temperature. Indeed, this periodicity in Euclidean time provides one (rather formal) way of understanding the origin of black hole radiance [24]. Sitting on top of each point in the plane is a two-sphere with radius r. The two-sphere with minimal radius $r = R_{BH} = 2M_{BH}$ sits above the origin of the plane.

We can compute the free energy F of a black hole in equilibrium with a radiation bath by using the Euclidean path integral method [23]. If the charge of the black hole is not specified, then F is given by

$$e^{-\beta F} = \int_\beta e^{-S_E/\hbar}, \tag{7.1}$$

where S_E denotes the Euclidean action, and the path integral is restricted to geometries with topology $R^2 \times S^2$ that are periodic in τ with period $\beta\hbar$. Hawking's black hole thermodynamics can be recovered in the limit $\hbar \to 0$ by expanding S_E about its saddle point, the Euclidean Schwarzschild solution.

If, however, we wish to compute the free energy $F(\beta, Q)$ in the charge-Q sector of a theory with manifest Z_N local symmetry, then the field configurations fall into distinct sectors that must be weighted by different phases. This is easiest to understand in the "thin-string" limit; that is, when the natural width μ^{-1} of the string is negligible compared to the size R_{BH} of the black hole. So, to begin with, let us consider this limit.

The path integral includes a sum over the world sheet of a (thin) cosmic string. But on a background with $R^2 \times S^2$ topology the world sheets are classified by a winding number k that specifies how many times the world sheet is wrapped around the two-sphere. Dependence of F on the charge Q arises because a world sheet with winding number k is weighted in the path integral by the Aharonov–Bohm phase $\exp(2\pi ikQ/N)$.* The action of the string is just the string tension T_{string} times the area of the world sheet. So it is evident that, among the $k = 1$ configurations, the one of lowest action is such that the world sheet is stretched around the two-sphere with the minimal radius R_{BH}. This configuration is the "world-sheet instanton" that dominates the Q-dependence of the free energy in the limit $\hbar \to 0$.

The presence of the string world sheet perturbs the background geometry, but this effect is small if the tension T_{string} is small in Planck units. So the string world sheet increases the action by

$$S_{string} = A_{BH} T_{string} = 4\pi R_{BH}^2 T_{string}, \tag{7.2}$$

and the one-instanton contribution to the partition function is of the form

$$(e^{-\beta F(\beta,Q)})_{one\text{-}instanton} \sim e^{-S_{Schwarzschild}/\hbar} e^{2\pi iQ/N} \exp\left[-A_{BH} T_{string}/\hbar\right], \tag{7.3}$$

where $S_{Schwarzschild}$ is the gravitational action of the Euclidean Schwarzschild geometry. This contribution is suppressed compared to the leading, Q-independent, term

$$(e^{-\beta F(\beta,Q)})_{zero\text{-}instanton} \sim e^{-S_{Schwarzschild}/\hbar} \tag{7.4}$$

by a factor that is exponentially small when $\hbar \to 0$, or when the surface area of the black hole is large compared to the inverse string tension.

By summing the contributions from one instanton and one anti-instanton, we find

$$\beta F(\beta, Q) - \beta F(\beta, Q = 0) \sim [1 - \cos(2\pi Q/N)] e^{-A_{BH} T_{string}/\hbar}. \tag{7.5}$$

This expression is independent of the volume of the radiation bath surrounding the black hole, and so should be regarded as a contribution to the intrinsic free energy of the black hole. From the thermodynamic identity

$$M(\beta, Q) = \frac{\partial}{\partial\beta}[\beta F(\beta, Q)], \tag{7.6}$$

we may extract the leading dependence on Q of the black hole temperature,

$$\beta(M_{BH}, Q) - \beta(M_{BH}, Q = 0) \sim [1 - \cos(2\pi Q/N)] e^{-16\pi M_{BH}^2 T_{string}}. \tag{7.7}$$

Given two black holes with the same mass, the one with larger Z_N charge is cooler. Quantum hair inhibits the emission of Hawking radiation, in a computable manner.

The Q-dependence of the temperature in eq. (7.7) is very weak in the thin-string limit, $A_{BH} T_{string} \gg 1$. But the Q-dependent contribution can be appreciable when

$$M_{BH} \sim T_{string}^{-1/2} \tag{7.8}$$

(in Planck units). If T_{string} is small in Planck units, then, the thermodynamic effects of quantum hair can become important while M_{BH} is still large in Planck units, so that it is reasonable to continue to neglect the effects of quantum gravity. In order to analyze the Q-dependence when the thin-string approximation is no longer valid, it is helpful to reformulate the topological classification of the matter field configurations in a more general way.

In this connection, we recall that the abelian Higgs model described in Section 2 contains instantons when it is formulated in flat *two-dimensional* spacetime. The instanton is just the magnetic vortex on R^2, with magnetic flux $2\pi/Ne$ trapped in its core. This instanton survives on a four-dimensional Euclidean background, *if* the background has the topology of $R^2 \times S^2$. In the thin-string limit, our world-sheet instanton may be re-interpreted as just such a vortex. The advantage of this interpretation is that the vortex-number classification of the field configurations can be extended to the regime in which the thin-string approximation no longer applies, and the action of the vortex is no longer large compared to $\hbar$.

By extending the analysis beyond the domain of validity of the thin-string approximation, one can hope to attain a quantitative understanding of how Z_N quantum hair turns off the Hawking evaporation of the black hole. Work on this problem is in progress [25].

An important lesson to be learned from the discussion above is that the no-hair principle carries less force on the Euclidean section of the black hole geometry than on the

* Formally, this phase is introduced into the path integral in order to project the sum over states, in the evaluation of the partition function $\mathrm{tr}(e^{-\beta H})$, onto the states with charge Q.

Lorentzian section. The Lorentzian black hole cannot support a stationary electromagnetic field, if the photon mass μ is nonvanishing. But the Euclidean section *can* support a vortex with nonzero flux. In the limit $\mu \to 0$, the flux of the vortex spreads out, and the Euclidean vortex solution smoothly approaches the Euclidean section of the electrically charged black hole. Thus, the $\mu \to 0$ limit is much less singular on the Euclidean section than on the Lorentzian section. Correspondingly, the $\mu \to 0$ limit is less singular in quantum physics than in classical physics.

8. Conclusions

The main point of this talk has been to demonstrate that a black hole can have Aharonov–Bohm "quantum hair" that influences its thermodynamic behavior. We have seen that this is the case, at least in a particular field-theoretic model. The existence of quantum hair exposes the limitations of the no-hair principle in the quantum domain.

However, I have not directly addressed an important question – Do black holes in *Nature* have quantum hair?

I don't know. But it is appropriate to note, in this connection, that our current ideas about physics at very short distances (particularly superstring theory) suggest that fundamental physics is governed by an enormous group of local symmetries.* Nearly all of these symmetries suffer spontaneous breakdown at or near the Planck scale. One is strongly tempted to speculate [5] that this symmetry breakdown gives rise to many varieties of quantum hair.†

Of course, even if quantum hair of the Aharanov–Bohm type does not exist in Nature, black holes may well carry quantum hair in a more general sense; that is, in order to give a complete quantum-mechanical description of processes involving black holes, it may be necessary to ascribe to the black hole many non-classical degrees of freedom. Quantum hair in this sense seems unavoidable if (contrary to Hawking's bold hypothesis) quantum coherence is actually maintained in processes involving black holes.

In any event, I expect that further investigation of models of quantum hair like that described in this talk will reward us with valuable insights into the quantum-mechanical behaviour of black holes, and hence into some of the deep mysteries of quantum gravity.

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Appendix: Quantum hair and axionic hair

I described in Section 2 how quantum hair can arise if a nontrivial local discrete symmetry survives when the Higgs mechanism occurs, and argued in Section 3 that a black hole can carry such quantum hair. In this appendix, I compare the black hole hair associated with a manifest local symmetry and another exotic type of black hole hair that has been proposed recently.

A theory in which a *global* U(1) symmetry is spontaneously broken contains an exactly massless Goldstone boson, the *axion*, and also a topological defect, the *axion string*. An axionic charge operator can be defined, and an object that carries this charge exhibits a nontrivial Aharonov–Bohm interaction with an axion string [27]. By means of this Aharonov–Bohm effect, axionic charge can in principle be detected at long range.‡ Thus, axionic charge is a type of hair that, according to the argument in Section 3, can be carried by a black hole. Black holes with axionic hair were first described by Bowick *et al.* [11].

Suppose that the spontaneous breakdown of the global U(1) symmetry is driven by the condensation of a scalar field

$$\phi = \frac{1}{\sqrt{2}}\rho e^{i\theta} \tag{A.1}$$

where

$$\langle \rho \rangle = v \tag{A.2}$$

and θ is the axion field. Then the axionic charge Q in a volume Ω may be expressed as

$$Q = v^2 \int_\Omega * \, d\theta, \tag{A.3}$$

where $*$ denotes the Hodge dual. The Aharonov–Bohm phase acquired by an axion string that winds around an object Q is

$$\exp(2\pi i Q). \tag{A.4}$$

Thus, it is only the non-integer part of Q that can be detected at long range by means of the Aharonov–Bohm effect, and only the non-integer part of Q that classifies the axionic hair on a black hole.

It is natural to suggest [11] that axionic wormholes [28] play a role in the evaporation of a black hole with axionic charge. However, the axionic charge that flows down a wormhole must be an integer [27–29]; wormhole processes therefore have no effect on the axionic hair. Since the non-integer part of Q can be detected at infinite range, it must be exactly conserved, in spite of wormholes or other exotica [5, 8, 30].

As Bowick *et al.* [11] noted, it is convenient to describe axionic charge using an alternative formalism, related to the above by a duality transformation. We may introduce a three-form field strength H defined by

$$H/2\pi = v^2 * d\theta. \tag{A.5}$$

This field strength can be expressed as the curl of a two-form potential

$$H = dB, \tag{A.6}$$

* See, for example, [26].

† It is far from clear, though, that the effects of such quantum hair could be reliably studied using semiclassical methods.

‡ I assume here that the axion mass is exactly zero. Effects of an axion mass are discussed in Ref. [8].

and the axionic charge in a volume Ω can be written as

$$Q = \frac{1}{2\pi}\int_\Omega H = \frac{1}{2\pi}\int_\Sigma B, \tag{A.7}$$

where Σ is the boundary of Ω.

The advantage of this dual formalism is that it allows us to discuss axionic hair using the language of classical field theory. According to the identity eq. (A.7), an object that carries axionic charge must have a long-range B field. Indeed, Bowick *et al.* [11] found that a black hole with axionic charge could be constructed as a solution to classical field equations. In this solution, the field strength H vanishes outside the event horizon, but the potential B is nonvanishing, as required by eq. (A.7).

Even though it admits this "classical" description, however, axionic hair should be regarded as a type of quantum hair. The B field is not a classical local observable; is not even gauge invariant.* The long-range B field of a black hole can be detected only by means of the Aharonov–Bohm interaction of the black hole with an axion string. This interaction is no more classical than is the Aharonov–Bohm interaction of a magnetic solenoid with an electrically charged particle, in massless electrodynamics.

In fact, even the Z_N quantum hair that results from the Higgs mechanism is amenable to this sort of "classical" description [25, 31]. A duality transformation similar to eq. (A.5) can be applied to the abelian Higgs model of Section 2. Then the three-form field strength is

$$H/2\pi = v^2 * (\mathrm{d}\theta + eA) \tag{A.8}$$

where θ is the phase of the Higgs field, and the electric charge may be expressed as in eq. (A.7).† In this dual version of the Higgs model, the Aharonov–Bohm interaction of a charge with a string arises from a term in the action of the model that resembles a Chern–Simons term; the Aharonov–Bohm phase is [25, 32]

$$\exp\left(\frac{ie}{2\pi}\int B \wedge F\right), \tag{A.9}$$

where $F = \mathrm{d}A$ is the electromagnetic field strength. To see that eq. (A.9) is just the Aharonov–Bohm phase, consider a history such that the world sheet of a cosmic string lies on a closed surface Σ, where the string carries magnetic flux $\Phi = 2\pi/e$. Then, if we neglect the thickness of the string, eq. (A.9) reduces to

$$\exp\left(\mathrm{i}\int_\Sigma B\right) = \exp(2\pi \mathrm{i} Q), \tag{A.10}$$

where Q is the charge enclosed by the world sheet. In the Higgs model of Section 2, this charge Q is an integer multiple of $1/N$, and eq. (A.10) is the Z_N Aharonov–Bohm phase.

By means of this dual reformulation of the abelian Higgs model, the black hole with Z_N quantum hair can be constructed as a solution to "classical" field equations [25, 31]. I emphasize once again, however, that in spite of the existence of this "classical" description, the Z_N charge is a type of quantum hair that escapes detection in the classical limit.

And the above discussion demonstrates that axionic hair may be regarded as a special case (the $e \to 0$ limit) of the more general quantum hair considered in this paper.

* The dual formulation respects the local symmetry $B \to B + \mathrm{d}\Lambda$, where Λ is a one-form. A gauge transformation that is nonsingular at the event horizon of a black hole can change the (unmeasurable) integer part of Q, but cannot change the (measurable) non-integer part.

† In a departure from the notation in Section 2, I have taken the charge of the Higgs field to be one.

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