Growing Hair on Black Holes

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A black hole can carry quantum numbers that are *not* associated with massless gauge fields, contrary to the spirit of the "no-hair" theorems. In the Higgs phase of a gauge theory, electric charge on a black hole generates a nonzero electric field outside the event horizon. This field is nonperturbative in $\hbar$ and is exponentially screened far from the hole. It arises from the cloud of virtual cosmic strings that surround the black hole. In the confinement phase, a magnetic charge on a black hole generates a *classical* field that is screened at long range by nonperturbative effects. Despite the sharp difference in their formal descriptions, the electric and magnetic cases are closely similar physically.

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The "no-hair" theorems [1] of black-hole physics state that a stationary black hole can be completely characterized by a few quantum numbers, namely, those associated with long-range gauge fields. These theorems give substance to the idea that, in the process of collapse to a black hole, anything that *can* be radiated away *will* be radiated away [2]. In the case of gravity coupled to a massless Abelian gauge field, the allowed quantum numbers are the mass M, the angular momentum J, and the (electric or magnetic) charge Q.

Models of elementary particle physics abound with *spontaneously broken* gauge symmetries. The charges corresponding to these symmetries are screened, and the vector mesons are massive. No-hair theorems can be extended to this case [3,4]; they indicate that, at the classical level, there is *no* sign of the broken gauge symmetry outside a black hole. In particular, there are no stationary black-hole solutions that are nonsingular at the event horizon and have a nonvanishing massive vector field that decays exponentially far from the hole [5]. Furthermore, outside the event horizon of a stationary magnetically charged black hole, the Higgs field is covariantly constant [5].

The symmetry breakdown may leave unbroken a residual discrete subgroup of the gauge group. Then, conserved charges can be defined [6–8], and such charges can reside on a black hole. Yet, according to the no-hair theorems, these charges have no *classical* effect on the properties of the hole. It would be quite disturbing if such charges were really dynamically impotent when carried by black holes, as the no-hair theorems suggest. For one thing, it would falsify the attractive idea [9] that sufficiently small black holes are fundamentally indistinguishable from sufficiently heavy elementary particles, since the latter certainly can carry these charges. It has been argued previously that the charge is observable in principle through Aharonov-Bohm scattering off suitable cosmic strings [6,10], but it would be disappointing if this rather exotic gedanken experiment were the whole story.

We argue here that a screened charge has quite tangible effects that can be detected without resorting to an experiment involving cosmic strings. In fact, there *is* an electric field external to a black hole carrying a screened electric charge; it could be detected by an experimenter armed with electroscopes and pith balls. Such locally measurable hair can actually be reconciled with the no-hair theorems in either of two ways. The theorems state that *classical* hair is incompatible with *classical* screening. We find that if the screening is classical (as in a weakly coupled gauge-Higgs system), then the charge can produce a nonclassical (electric) field that is nonperturbative in $\hbar$. On the other hand, if the screening is quantum mechanical (as in a strongly coupled confining gauge theory), then the charge can produce a classical (magnetic) field that is independent of $\hbar$. Electric fields are generated by a process in which a virtual loop of cosmic string lassoes the hole and then reannihilates. Similarly, magnetic fields arise from the cloud of virtual electric-flux tubes that surrounds the hole.

Screened electric fields.—We focus on a simple model with a Z_N gauge symmetry, an Abelian gauge theory in which a scalar with charge $N\hbar e$ condenses, where $\hbar e$ is the charge quantum of the theory. The charge modulo N on a black hole can be detected by means of the Aharonov-Bohm interaction of the hole with a cosmic string [6]. But the no-hair theorem ensures that the charge has no classical effect on the black hole, nor is there any effect on small fluctuations of the quantum fields about the classical background. The dynamical effects of the charge are nonperturbative in $\hbar$; they are associated with *large* quantum fluctuations—virtual loops of cosmic string [11,12].

Nonperturbative effects are most conveniently studied using Euclidean path-integral methods. The black-hole contribution to the partition function at temperature β^{-1} is found [13] by summing over configurations that are periodic in imaginary time τ with period $\beta\hbar$, have the topology $R^2 \times S^2$, and are asymptotically flat; the sum is weighted by $e^{-S/\hbar}$, where S is the Euclidean action. The saddle points of this integral are the (Euclidean) classical black-hole solutions. In the Euclidean Schwarzschild solution, the Schwarzschild coordinates r and τ are polar coordinates on R^2, and the two-sphere of minimal radius $r=2M$ sits above the origin of the plane—this minimal sphere is the Euclidean vestige of the event horizon.

If we are interested in the behavior of black holes with a definite value of the Z_N charge, we must insert a projection operator into the path integral. The expression for the partition function of a black hole with charge Q in equilibrium with a radiation bath at temperature β^{-1} then becomes

$$Z_Q(\beta) = \sum_{k=-\infty}^{\infty} \exp\left(-i\frac{2\pi k Q}{N\hbar e}\right) Z_k(\beta)\,, \tag{1}$$

where $Z_k(\beta)$ is the path integral over periodic configurations that satisfy the constraint

$$e\int_0^{\beta\hbar} d\tau\, A_\tau(\tau,r) \underset{r=\infty}{\longrightarrow} \frac{2\pi}{N}k\,. \tag{2}$$

Equation (2) has a remarkable interpretation. If we regard $F_{r\tau}$ as a *magnetic* field, then $\int_0^{\beta\hbar} d\tau\, A_\tau$ is the magnetic flux in the r-τ plane, and k is the *vorticity*, the value of the flux in units of the flux quantum $2\pi/Ne$. The electric charge Q tells us with what phase to weight the different vorticity sectors.

In the semiclassical limit ($\hbar \to 0$, with $\beta\hbar$ fixed), the Q dependence of the partition function is dominated by the configurations with nonzero vorticity that have the lowest action; these are the classical vortex solutions with $k=\pm 1$. The action of the vortex can be computed analytically in two different limiting cases. As the natural thickness of a cosmic string in flat space becomes small compared to the size of the black hole (the "thin-string limit"), the vortex shrinks to a point at the origin of the r-τ plane (the location of the minimal two-sphere). If the string tension is small in Planck units, the vortex perturbs the background spacetime only slightly. The action of the $k=1$ solution is $S_{k=1} = S_{\text{gravity}} + \Delta S_{\text{vortex}}$; here $S_{\text{gravity}} = (\beta\hbar)^2/16\pi$ is the action of the Euclidean Schwarzschild solution, and $\Delta S_{\text{vortex}} = A_{\text{BH}} T_{\text{string}}$, where A_{BH} is the area of the event horizon and T_{string} is the tension of the minimal cosmic string. In the opposite ("thick-string") limit, the Higgs field contribution to the action can be neglected, and the vortex is well approximated by a Coulombic configuration with "charge" $q=2\pi/Ne$; the action is $S_{\text{gravity}} + \Delta S_{\text{vortex}}$, where $\Delta S_{\text{vortex}} = (\beta\hbar)\frac{1}{2}q^2/4\pi R = 2\pi^2/(Ne)^2$. Again, we have assumed that the back reaction of the vortex on the metric is small; this is a good approximation provided $R \gg [\hbar(Ne)^2]^{-1/2} R_{\text{Planck}}$. (Neglecting back reaction will simplify the analysis described below. Because the configurations that we sum over all have the same four-geometry, it is trivial to compute expectation values at a fixed point in spacetime. If we needed to sum over configurations with different geometries, things would be much more complicated.)

The leading charge-dependent contributions to the expectation values of static observables on the black-hole background can also be computed by summing contributions from the $k=\pm 1$ sectors, weighted by $\exp(-i2\pi Qk/N\hbar e)$. Two operators of particular interest are the electric field and the energy-momentum associated with it. The vortex and antivortex have equal and opposite $F_{r\tau}$; their contributions would cancel if the sectors were equally weighted. But if the charge Q on the black hole is nonzero, vortex and antivortex are weighted by unequal phases, and the Euclidean electric field acquires an expectation value proportional to $2i\sin(2\pi Q/N\hbar e)$. Continuing to real time, we find the electric field

$$\langle E(r)\rangle = 2\sin(2\pi Q/N\hbar e)C(\beta\hbar) \times\exp(-\Delta S_{\text{vortex}}/\hbar)[F_{r\tau}(r)]_{\text{vortex}}\,, \tag{3}$$

where $C(\beta\hbar)$ is a ratio of functional determinants arising from the integral over small fluctuations about the classical solution. The field strength $\langle E(r)\rangle$ falls off exponentially as $e^{-\mu r}/r$, for $r \gg \mu^{-1}$ (where $\hbar\mu$ is the mass of the vector meson). An expression similar to Eq. (3) is found for $\langle E^2\rangle$ (and hence for the expectation value of energy-momentum), but with $\cos(2\pi Q/N\hbar e)$ replacing $\sin(2\pi Q/N\hbar e)$. Note that it is far from true that the expectation value of the square is equal to the square of the expectation value—not only is the charge dependence different, but also the *same* exponential tunneling factor appears (not the square). Indeed, the same exponential factor appears in the expectation value of any power of E. These are not the moments of a probability distribution in which all measurements find an exponentially small field. Rather, most measurements find no field, but exponentially rare measurements find a field of order 1. Of course, this is typical of tunneling processes. If we measure the momentum density at some point outside a radioactive nucleus, most measurements find nothing, but on rare occasions an α particle is passing by. Knowledge of the Z_N charge tunnels out from behind the event horizon just as the α particle tunnels out from behind the potential barrier.

There is a very appealing spacetime picture corresponding to the calculation we have just described. If an object carries Z_N charge, then processes in which the object passes on either side of a cosmic string are weighted by different Aharonov-Bohm phases. Even in the absence of actual strings, the Aharonov-Bohm phase modulates the amplitude for *virtual* processes in which a string world sheet envelops the charge. Consider, then, a virtual process in which a loop of cosmic string nucleates at a point on the event horizon of a black hole, sweeps around the horizon two-sphere, then shrinks and annihilates at the antipodal point. The virtual string has magnetic flux in its core; hence its motion creates an electric field orthogonal to the magnetic field and the direction of motion, an electric field in the radial direction. The time-integrated value of this radial field is purely geometrical—although the electric field is proportional to the velocity, the time that the string spends at any point on the sphere is inversely proportional to the velocity. We must also average over all possible points of nucleation. This averaging cancels the magnetic field of the string, but the radial electric field survives. The vortex described above may be interpreted as this averaged string world sheet, and the phase in Eq. (1) is just the Aharonov-Bohm phase associated with a world sheet that wraps around the black hole k times.

The vortex is a solution to the *Euclidean* classical field equations that is nonsingular and has hair, e.g., a nonzero electric field. The existence of this solution is consistent with the no-hair theorems because when continued back to real time, the solution fails to satisfy suitable reality conditions—the electric field becomes imaginary. On the other hand, expectation values of observables, obtained by summing over the vorticity k, are real, but do not satisfy the classical field equations. (These equations are nonlinear, and we have seen that the expectation value of a product is not a product of expectation values.) One should also note that the vortex solution is not static, for it is transformed under time reversal into an antivortex. Yet the expectation values are static. In short, by performing the sum over k, we proceed from nonstatic solutions to static nonsolutions, thus violating the spirit of the no-hair theorems while respecting their mathematical content. (This mechanism is quite distinct from the idea that the broken symmetry is restored near the horizon [14], an idea for which we find no evidence.)

To avoid misunderstanding, we should add a clarifying remark. The Z_N hair that we have described endows the black hole with a new quantum number; it enlarges the black-hole state space. Such hair might be called *primary* hair, to distinguish it from another type—*secondary* hair, or hair growing on hair. Recently discussed examples of secondary hair are dilaton hair on electrically (or magnetically) charged black holes [15,16] and axion hair on dyonic black holes [16,17]. In these examples, the dilaton or axion has a nonvanishing expectation value outside the horizon. But the expectation value is *not* associated with a new quantum number, because it is completely determined by the values of conserved gauge charges. Such secondary hair occurs even in the standard model. Radiative corrections in the standard model induce higher-dimension terms in the effective action that couple matter fields or gauge fields to curvature. These terms can, for example, excite a (dipole) electric field outside a rotating black hole, or induce secondary Higgs field hair on a nonrotating hole.

Screened magnetic fields.—If a gauge theory is in a Higgs phase, electric fields are screened, and magnetic fields are confined to flux tubes. These phenomena can be described classically—the screening length and flux quantum are independent of $\hbar$. If a gauge theory is in a confinement phase, magnetic fields are screened, and electric fields are confined to flux tubes. These phenomena are quantum mechanical; in fact, the inverse screening length is nonperturbative in $\hbar$. In spite of this formal distinction, the Higgs and confinement phases have very similar physical properties.

We have seen that Z_N electric charges can be introduced into a Higgs theory, such that the charges have an Aharonov-Bohm interaction with a magnetic-flux tube. Likewise, an SU(N) gauge theory admits Z_N magnetic monopoles as point defects. The Z_N magnetic charges have an Aharonov-Bohm interaction with an electric-flux tube; these charges can be carried by black holes [7,12]. If SU(N) is the unbroken subgroup of an appropriately chosen larger spontaneously broken group, these monopoles may appear as topological solitons. But classical black-hole solutions with Z_N magnetic charges can be constructed in any case, just as black holes with magnetic charges can be constructed in ordinary electrodynamics.

The limit in which the confinement length scale is small compared to the size of the black hole is similar to the thin-string limit considered above—a moving virtual electric-flux tube that lassoes the hole generates a magnetic field. [More precisely, the gauge-invariant operator $\mathrm{tr}(F_{\theta\phi}^2)$ acquires an expectation value that depends on the magnetic charge of the black hole.] This field falls off exponentially, and is suppressed by the factor $\exp(-A_{\mathrm{BH}}T_{\mathrm{string}}/\hbar)$. The opposite (thick-string) limit is rather different from before. Now we find that the path integral is dominated by a classical solution with a nonzero magnetic field outside the horizon; hence, the expectation value of the field near the horizon is $\hbar$ independent. (Quantum effects still screen the field at long range.) No-hair theorems concern solutions to the classical field equations. They do not forbid this type of magnetic hair, because the screening of the field does not enter into a classical analysis.

This difference between the behavior of electric and magnetic Z_N hair in the thick-string limit arises because of the different role played by $\hbar$ in the two cases. The electric charge quantum is of order $\hbar$, while the magnetic charge quantum is $\hbar$ independent. Correspondingly, the flux carried by a magnetic vortex is $\hbar$ independent, while that carried by an electric vortex is of order $\hbar$. Thus, a cosmic string is a classical object, and the effects of virtual strings are heavily suppressed, while an electric-flux tube is a quantum-mechanical object that occurs copiously in quantum fluctuations. The difference is sharpest if we demand that both the Higgs theory and the confining theory are weakly coupled at the event horizon, so that semiclassical methods are applicable in both cases. The difference blurs if the confinement distance scale becomes comparable to the size of the black hole (so that virtual electric-flux tubes that envelop the hole become suppressed), or if the Higgs theory becomes strongly coupled (so that virtual cosmic strings become unsuppressed).

Other quantities, other holes.— We have found that a black hole can carry quantum numbers that are *not* associated with massless gauge fields, contrary to what the no-hair theorems may seem to suggest. A black hole endowed with such quantum numbers has locally measurable hair— nonvanishing fields outside the horizon that are screened at long range.

Such screened hair also affects the thermal [18] behavior of black holes, as we have discussed elsewhere [12]. Charge on a black hole reduces its temperature, compared to an uncharged hole with the same mass. The effect is exponentially small for screened electric charge in a weakly coupled theory, but can be quite significant for screened magnetic charge; there are zero-temperature black holes (analogous to the extreme Reissner-Nordström holes) that are stabilized by fields that are essentially invisible (exponentially decaying) at large distances.

In the classical analysis of the field equations [2], linear perturbation theory suggests that a massless (integer) spin s field can support hair in partial waves $l \leq s-1$. Associated with the spin-2 graviton, then, we have hair in partial waves $l=0$ (namely, M) and $l=1$ (J); associated with the spin-1 photon, we have hair in $l=0$ (Q). We have shown that the restriction to massless fields can be removed in the case $s=1$. It will be interesting to see whether this result can be extended to symmetries associated with higher spins, such as are suggested by superstring theory.

In a realistic model, various discrete gauge charges, as well as ordinary electromagnetic charge, might be relevant to the structure of extreme (zero-temperature) black holes. It may be significant in this regard that for the recently discovered charged dilaton black holes, the radius approaches zero as the extreme family is approached [15,16]. Therefore, short-distance physics, including the effects of virtual strings discussed here, becomes especially important for the description of the extreme holes. If we are ever lucky enough to find a small stable black hole (perhaps a magnetic monopole) left over from the big bang, its detailed properties will reflect this physics.

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