WORMHOLES IN SPACETIME AND θ_{QCD} ☆

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We calculate in chiral perturbation theory the dependence of Newton's gravitational constant G on the θ parameter of quantum chromodynamics, and we find that G, as a function of θ, is minimized at $\theta \simeq \pi$. This calculation suggests that quantum fluctuations in the topology of spacetime would cause θ to assume a value very near π, contrary to the phenomenological evidence indicating that θ is actually near 0.

Wormholes are quantum fluctuations in the topology of spacetime. If such fluctuations occur in quantum gravity, their consequences may be profound. It has been proposed that, because of wormhole effects, the fundamental constants of nature are afflicted by an intrinsic quantum indeterminacy; we must regard our universe as having been chosen at random from an ensemble of possible universes, all with different values of the fundamental constants [1–3]. Coleman has proposed a form for the probability distribution of possible universes [4]. The different universes may be labeled by a set of parameters, denoted α, that are presumably infinite in number; all fundamental constants are functions of α. According to Coleman's proposal, the probability $\mathrm{d}P$ that a randomly selected universe has a label in the interval between α and $\alpha+\mathrm{d}\alpha$ is

$$\mathrm{d}P=\mathrm{d}\alpha\, f(\alpha)\, \exp\{\exp[3/8G^2(\alpha)\Lambda(\alpha)]\}\,. \qquad (1)$$

Here Λ is the cosmological constant, or vacuum energy, and G is Newton's gravitational constant; both are functions of α. The function f is a smooth function of α whose detailed properties need not concern us.

As Coleman observed, the distribution eq. (1) is very sharply peaked on the surface in α-space where $\Lambda(\alpha)=0$. This observation may explain why, in nature, the cosmological constant is indeed found to be very small. In fact, by regulating this distribution in a plausible way as Λ approaches zero, one finds that it strongly favors not only that $\Lambda(\alpha)=0$, but also that $G(\alpha)$ assume the smallest possible value on the surface in α-space where $\Lambda(\alpha)=0$ [5,6]. The requirement that $G(\alpha)$ is at its minimum on the $\Lambda=0$ surface is likely to determine all of the remaining α's, and hence to fix all other constants of nature. If the possible universes are subject to the probability distribution eq. (1), then, the quantum indeterminacy of fundamental physics turns out to be very mild. For each fundamental constant there is a "standard" value that is in principle calculable, and the probability is unity that in a randomly selected universe each constant assumes its standard value.

(Actually, the distribution eq. (1) is not universally accepted [7,8]. Hawking [7], for example, suggests that the correct behavior of the probability distribution for Λ near zero is $\mathrm{d}P\sim \mathrm{d}\alpha\, \exp(3/8G^2\Lambda)$, rather than the double exponential in eq. (1). But the conclusion that G assumes, with probability equal to unity, its minimum value on the surface $\Lambda=0$ would apply also in this case.)

While the constants of nature may be determined in principle by the distribution eq. (1), the values of constants other than the cosmological constant cannot be easily computed. To determine other parameters that characterize low-energy physics, we must minimize G as a function of these parameters. But the functional dependence of G on other couplings of

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interest is typically very sensitive to the details of physics at the Planck scale; it cannot be computed in terms of low-energy physics alone [6].

Our main purpose in this letter is to point out an exception to the general rule annunciated above. The dependence of G on the θ-parameter of quantum chromodynamics [9] *can* be computed in terms of low-energy physics alone. The point is that θ-dependence arises only through nonperturbative strong interaction effects, and these are presumably exponentially small at the Planck scale, because of asymptotic freedom. We have calculated the θ-dependence of G in an approximation that is valid if the masses of the light up and down quarks are sufficiently small. We find, assuming the validity of this approximation, that the minimum of G occurs for θ very near π. (The minimum would be at exactly $\theta=\pi$ were it not for small CP-violating effects due to the weak interactions.)

We therefore expect that $\theta\simeq\pi$ is overwhelmingly favored by the probability distribution eq. (1). Since the phenomenological evidence suggests that instead $\theta\simeq 0$, our calculation indicates a potentially serious conflict between current ideas about wormholes and observed low-energy physics. We will comment further below about how this conflict might be resolved.

Because the θ parameter is CP odd, and the strong interactions conserve CP to remarkable accuracy, it has long been recognized that θ must be extremely close to either 0 or π. ($\theta=\pi$ is a CP-conserving value because θ is a periodic variable defined modulo 2π.) The experimental limit on the electric dipole moment of the neutron indicates that θ deviates from 0 or π by an amount at most of order 10^{-9} [10]. Given that large CP-violating phases do infect the weak interactions, this inclination of the strong interaction to conserve CP poses a serious puzzle. The most satisfying explanation for the CP conservation by the strong interaction is that originally suggested by Peccei and Quinn [11]. They proposed that θ is actually a dynamical variable, and therefore assumes that value that minimizes the energy density of the vacuum. A powerful nonperturbative argument shows that the vacuum energy density of QCD, as a function of θ, is minimized at $\theta=0$ [12]. Thus, the Peccei–Quinn mechanism naturally explains why θ is very close to the CP-conserving value. (The minimum of the energy density is perturbed slightly away from $\theta=0$ by CP-violating effects due to the weak interaction. The amount of the perturbation depends on the detailed nature of CP violation; in the Kobayashi–Maskawa model, one can estimate that the minimum occurs for $\theta\sim 10^{-14}$, which is well within the experimental limit.) Another interesting consequence of this mechanism is that there exists a very light, very weakly interacting particle, the axion, associated with the oscillations of θ about the minimum [13,14]. It has even been proposed that these axions comprise the dark matter of the universe [15].

With the context of wormhole physics, the Peccei–Quinn explanation for $\theta\simeq 0$ is problematic. Their mechanism relies on the existence of an approximate global symmetry, the Peccei–Quinn symmetry, that is intrinsically broken only by a color anomaly. But wormholes have no respect for global symmetries (whether exact or approximate). Rather, wormhole effects are expected to generate α-dependent couplings of all types consistent with the *local* symmetries of fundamental physics [1,2]. It will not do, then, to invoke a Peccei–Quinn symmetry by fiat; the symmetry itself requires an explanation.

(A similar remark applies to another explanation that is sometimes proposed for the small value of the electric dipole moment of the neutron – that the mass of the up quark is zero, or very close to zero. This is no explanation unless one understands *why* the up quark is massless. Indeed, this proposal is closely related to the Peccei–Quinn mechanism, for if the up quark is massless, then there is an approximate global symmetry that is intrinsically broken only by a color anomaly.)

In spite of the above comments, wormholes and the Peccei–Quinn mechanism might be reconcilable. Two possibilities come to mind. Perhaps an approximate Peccei–Quinn mechanism arises in low-energy physics as an accidental consequence of local symmetries, which are not disturbed by wormhole effects. (This would be like the approximate conservation of baryon number in the standard model that is an automatic consequence of local $SU(3)\times SU(2)\times U(1)$ invariance.) It is not so easy to make this idea workable, however. The problem is that it does not suffice for the accidental Peccei–Quinn symmetry to apply to the operators in the effective action that are of renormalizable type (dimension four or less). If the Peccei–Quinn mechanism is to ensure that θ is very small, then nonperturbative strong interaction effects must swamp all other effects that break the Peccei–Quinn symmetry; this constraint typically requires that the symmetry be satisfied by operators of quite high dimension [16]. There is another possible way to rescue the Peccei–Quinn mechanism, in spite of the tendency of wormhole effects to break global symmetries. Although the Peccei–Quinn symmetry is badly broken for generic values of α, it may become a good approximate symmetry for that particular "standard" value of α that minimizes G.

At any rate, there appears to be ample motivation to consider whether, within the context of wormhole physics, the CP conservation of the strong interactions can be explained without appealing to the Peccei–Quinn mechanism, and without requiring the existence of a light axion. Indeed, the crucial feature of the Peccei–Quinn mechanism is that it makes θ an adjustable quantity, a dynamical variable that seeks the minimum of the energy density at $\theta=0$. And wormhole effects also make θ an adjustable quantity, not a dynamical variable, but an α-dependent coupling constant that seeks the sharp peak in the probability distribution eq. (1). Furthermore, as Nielsen and Ninomiya [17] recently stressed, G is CP even while θ is CP odd; therefore strong interaction effects generate a dependence of G on θ that is an even function of θ. This function is stationary at both $\theta=0$ and $\theta=\pi$, and so it is reasonable to expect that its minimum occurs either at $\theta=0$ or at $\theta=\pi$. Since the peak in the probability distribution occurs where G is minimized, the CP conservation by the strong interactions is naturally explained. (As for the Peccei–Quinn mechanism, CP-violating effects due to the weak interventions perturb the minimum, but only slightly.)

The dependence of θ on α arises as follows: The Yukawa couplings of the quarks to the Higgs doublet are modified by wormhole effects, and hence are α-dependent in both modulus and phase. When the electroweak gauge symmetry is spontaneously broken, this α-dependence enters the quark mass matrix. Some of the phases in the mass matrix are unobservable, because they can be removed by a redefinition of the phases of the quark fields. But there remain, as observable parameters, the values of the quark masses and the Kobayashi–Maskawa angles and phases that infect the charged weak current. Finally, there is one phase that can be removed from the quark mass matrix only by a field redefinition that has a color anomaly. This phase is θ. It is irrelevant in all orders of perturbation theory, but nonperturbative strong interactions do depend on θ.

We will assume in the ensuing discussion that it is possible to adjust the α-parameters so that θ changes, while all other couplings in the effective lagrangian remain fixed. It is easy to construct toy models that behave this way, and we expect that this behavior is reasonably generic. When the α-dependence of Newtons's constant G is considered, one finds that perturbative renormalization effects induce large contributions to G that depend on the quark masses and the KM angles. These contributions are of order $M_{\rm Pl}^2$, where $M_{\rm pl}$ is the Planck mass scale, and are sensitive to the details of Planck-scale physics. The criterion that $G(\alpha)$ is at its minimum on the surface $\Lambda(\alpha)=0$, then, determines these quantities, but only in a manner that cannot be computed based on a knowledge of low-energy physics alone [6]. But since the dependence of G on θ arises only from nonperturbative strong interaction effects, θ is calculable based on low-energy physics alone, at least in principle.

Before we proceed with our calculation of $G(\theta)$, one more point needs emphasis. We asserted above that θ can be determined by finding the minimum of $G(\theta)$, but the actual criterion that determines the constants of nature is that $G(\alpha)$ is minimized on the surface where $\Lambda(\alpha)=0$. We must explain why it is an excellent approximation to disregard the requirement that $\Lambda(\alpha)=0$. The crucial point is that the dependence of Λ and G on θ is characterized by the strong interaction scale, rather than the Planck scale. If we perturb θ by a small amount $\delta\theta$, Λ and G change according to

$$\delta\Lambda=a(\theta)\delta\theta, \quad \delta(1/16\pi G)=b(\theta)\delta\theta\,; \tag{2}$$

we will calculate $a(\theta)$ and $b(\theta)$ below, in chiral perturbation theory. But when a generic α-parameter is perturbed by $\delta\alpha$, we have instead, schematically,

$$\delta\Lambda\sim M_{\rm Pl}^4\,\delta\alpha, \quad \delta(1/16\pi G)\sim M_{\rm Pl}^2\,\delta\alpha\,. \tag{3}$$

Thus, if we perturb θ and adjust α slightly to remain on the $\Lambda=0$ surface, the change of G is given by

$$\delta(1/16\pi G)|_{\delta\Lambda=0}\sim[b(\theta)-a(\theta)/M_{\rm Pl}^2]\,\delta\theta\,. \tag{4}$$

Because a and b are very small in Planck units, the second term in eq. (4) is negligible. We may just as well minimize $G(\theta)$ without regard for the $\Lambda=0$ constraint.

Now we are finally prepared to describe the calculation of $G(\theta)$, in chiral perturbation theory. The main idea that underlies the calculation is quite simple. If the pion mass were very small, as would be true if the up and down quarks were sufficiently light, then the strong interaction contribution to G would be dominated by a one-pion-loop diagram that has a calculable logarithmic sensitivity to m_π^2. Then, when the pion is light enough, the dependence of G on θ can be calculated from the dependence of m_π^2 on θ. One finds that $G(\theta)$ is minimized when $m_\pi^2(\theta)$ is minimized. And it is easy to see, again in the limit where the pion is sufficiently light, that $m_\pi^2(\theta)$ is minimized at $\theta=\pi$.

To perform the calculation, we make use of a chiral lagrangian that describes the self-interactions at low momenta of the pseudo-Goldstone bosons π^+, π^-, π^0. This chiral lagrangian respects a nonlinearly realized $SU(2)_L \times SU(2)_R$ chiral symmetry. It can be expressed in terms of a field $\Sigma(x)$ that is a 2×2 unitary matrix with determinant one, and that transforms under chiral symmetry as

$$\Sigma \to V_L \Sigma(x) V_R^\dagger , \tag{5}$$

where $V_L \in SU(2)_L$ and $V_R \in SU(2)_R$. In terms of the pion fields, Σ can be expressed as

$$\Sigma = \exp(2i\Pi/f) ,$$
$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} , \tag{6}$$

where f is the pion decay constant. The chiral lagrangian can be expanded in powers of the derivatives acting on the Σ field; terms with more derivatives are suppressed at low energy by additional powers of the pion momentum.

The effects of the explicit breaking of chiral symmetry by quark masses can also be systematically incorporated in the chiral lagrangian. If m is the 2×2 mass matrix of the light quarks, then QCD respects a formal symmetry in which eq. (5) is accompanied by

$$m \to V_R m V_L^\dagger . \tag{7}$$

By demanding invariance under this formal symmetry, we find that the leading mass-dependent terms in the chiral lagrangian, in a curved spacetime background, are

$$\mathscr{L}_{\text{mass}} = v\,\text{tr}(m\Sigma + \Sigma^\dagger m^\dagger)(1+cR) + \dots . \tag{8}$$

Here v is a quantity with the dimensions of $(\text{mass})^3$, and c is a quantity with the dimensions of $(\text{mass})^{-2}$; both are determined by nonperturbative strong interaction effects. R is the curvative scalar of the background spacetime. In eq. (8), we have neglected terms that contain derivatives of the Σ field, more powers of the light quark mass matrix m, or more powers of the curvature R.

The θ-parameter enters the chiral lagrangian in the light quark mass matrix m, through the relation

$$\theta = \arg(\det m) . \tag{9}$$

We find the precise form of m by performing a chiral rotation of m that ensures that $\mathscr{L}_{\text{mass}}$ contains no "tadpole" terms linear in the pion fields. The result is

$$m = \begin{pmatrix} m_u e^{i\phi} & 0 \\ 0 & m_d e^{i(\theta-\phi)} \end{pmatrix} , \tag{10}$$

where

$$\sin\phi = \frac{m_d \sin\theta}{(m_u^2 + m_d^2 + 2m_u m_d \cos\theta)^{1/2}} ,$$

$$\sin(\theta-\phi) = \frac{m_u}{m_d} \sin\phi . \tag{11}$$

By expanding Σ in powers of the pion field, we find, in tree approximation in the chiral lagrangian and to lowest order in light quark masses,

$$m_\pi^2(\theta) = (4v/f^2)(m_u^2 + m_d^2 + 2m_u m_d \cos\theta)^{1/2} ,$$
$$\Lambda(\theta) = \Lambda_0 - \tfrac{1}{2} f^2 m_\pi^2(\theta) ,$$
$$1/16\pi G = 1/16\pi G_0 - \tfrac{1}{2} c f^2 m_\pi^2(\theta) \tag{12}$$

here Λ_0 and G_0 are constants independent of θ.

Arguments based on QCD inequalities show that v is nonnegative [12]. Hence, the vacuum energy is evidently minimized at $\theta=0$, as is required for the Peccei–Quinn mechanism to work. But the expression for G in eq. (12) could be minimized at either $\theta=0$ or $\theta=\pi$, depending on the sign of c. Though the sign of c is determined in principle by the nonperturbative strong interactions, we do not know how to compute it reliably. Nonetheless, what we have found is consistent with the expectation of Nielsen and Ninomiya, that the minimum of G occurs at a *CP*-conserving value of θ.

In fact, it is possible to go further, because the tree approximation contribution to the θ-dependence of $(16\pi G)^{-1}$ in eq. (12) is not actually the leading contribution when m_π^2 is very small. There is a contribution from one pion loop that is enhanced by a logarithm of the pion mass. This logarithmically enhanced contribution is [18,5,6]

$$\delta(1/16\pi G)$$
$$= -(1/64\pi^2) m_\pi^2(\theta) \ln[M_{CSB}^2/m_\pi^2(\theta)] , \qquad (13)$$

where M_{CSB} is the "chiral symmetry breaking scale" of QCD; a naive estimate of it is $M_{CSB} \sim 4\pi f \sim 1$ GeV. (Eq. (13) is the one-loop contribution to $(16\pi G)^{-1}$ that arises from the minimal coupling of the pion to gravity. There is also a one-loop contribution that involves the nonminimal coupling of the pion to R in eq. (8), but this contribution is of order $(m_\pi^4 \ln m_\pi^2)$ and hence higher order in chiral perturbation theory.)

If the pion mass is sufficiently small, then the one-loop contribution to $(16\pi G)^{-1}$ in eq. (13) dominates the tree contribution in eq. (12). The calculated sign of the one-loop contribution shows that $m_\pi^2(\theta)$ seeks the smallest possible value in order to minimize $G(\theta)$. In view of the expression for $m_\pi^2(\theta)$ in eq. (12), this means that $\theta=\pi$ is the preferred value. We have shown, then, that at least in a world in which the light quark masses are sufficiently small, the criterion that $G(\alpha)$ is at its minimal value on the surface in α-space where $\Lambda(\alpha)=0$ requires θ to be very close to π. (As in the Peccei–Quinn model, weak interactions perturb θ slightly away from the value chosen by QCD, by an amount of order 10^{-14} in the KM model of *CP* violation.)

The approximation of neglecting the contribution to $(16\pi G)^{-1}$ in eq. (12) compared to the contribution in eq. (13) is justified provided that

$$(1/32\pi^2 c f^2) \ln(M_{CSB}^2/m_\pi^2) \gg 1 , \qquad (14)$$

it is not clear whether it is justified for realistic values of the light quark masses. To get some insight about whether the conclusion that G is minimized at $\theta=\pi$ survives beyond the approximation of very light quark masses, we have considered the opposite limit of infinite quark masses, or pure Yang–Mills theory. In pure Yang–Mills theory, we have computed $G(\theta)$ in the dilute instanton gas approximation. Unlike chiral perturbation theory, which can be justified when the quark masses are small enough, the dilute instanton gas approximation cannot really be justified. Nonetheless, it is known to give the right answer for the vacuum energy; namely, that the minimum occurs at $\theta=0$, in agreement with the QCD inequality argument.

To calculate $G(\theta)$ we compute the connected two-point function of the energy–momentum tensor and extract its leading behavior at low momentum. The calculation turns out to involve a subtlety concerning the trace anomaly in the presence of instantons; we will not report on the details here. The result is that the minimum of $G(\theta)$ occurs at $\theta=\pi$. Thus, the dilute instanton gas calculation lends support to the view that $G(\theta)$ is minimized at $\theta=\pi$ generically, irrespective of the value of quark masses. Perhaps it will eventually be possible to resolve this issue by doing numerical calculations in lattice QCD.

Finally, let us consider whether our conclusion that $\theta \simeq \pi$ is in conflict with experiment. There is suggestive evidence that θ is actually close to zero in nature [10]. But one should recall that this evidence is based on chiral perturbation theory calculations of the pseudoscalar meson masses that treat the *strange* quark mass as a small parameter, a somewhat dubious procedure [19]. If the corrections to leading order perturbation theory in the strange quark mass turn out to be surprisingly large, then it may be that θ is really close to π in nature after all, as wormhole considerations indicate. Again, this issue may ultimately be resolved by lattice QCD calculations.

To summarize, we have argued that, at least in an approximation in which the masses of the up and down quarks are taken to be very small, wormhole fluctuations in the topology of spacetime drive the θ parameter of QCD to $\theta \simeq \pi$. Since $\theta \simeq 0$ appears to be satisfied in nature, this prediction poses a possible conflict between wormhole physics and experiment. We have noted several ways in which this conflict might be resolved. Perhaps a Peccei–Quinn symmetry can survive in spite of wormhole effects, allowing θ to relax dynamically to the value $\theta=0$. Perhaps chiral perturbation theory is misleading, and wormholes actually prefer $\theta \simeq 0$ for realistic values of the light quark masses. And finally, it is at least conceivable that $\theta \simeq \pi$ really is satisfied in nature, in accord with our prediction.

After completion of this work we found that Choi and Holman have also concluded (using different methods) that wormholes favor $\theta = \pi$ [20].

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