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FRACTIONAL CHARGE AND MAGNETIC MONOPOLES

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1. Introduction

Last spring, when word began to circulate that Blas Cabrera had detected something interesting, many of us recognized the opportunity to make an ironic remark (see, for example, Ref. 1-3): Two experiments, both performed at Stanford, had obtained results which were flatly contradictory. Cabrera[4] had observed a candidate magnetic monopole carrying the Dirac magnetic charge $g_D = 1/2e$. Fairbank and collaborators[5] had observed fractional electric charge $q = e/3$ on matter. Together, these two observations violate a very general consistency condition derived many years ago by Dirac.[6]

I first met Bill Fairbank at the Vanderbilt Conference on Particle Physics last May, and could not resist the temptation to make this ironic remark. Although, of course, he had heard it before, Bill responded kindly and patiently. He suggested that, instead of trying to be ironic, I should give some serious thought to the possibility that both results are correct. They are good experiments. What if both are right? This struck me as a provocative question.

So I thought about it a little.[7] Others[8-11] have done the same, presumably without any prompting from Bill Fairbank. For the remainder of this talk, I will assume without further apology that both experiments are correct; that is, I will assume that Cabrera has really discovered a monopole with the Dirac magnetic charge, and that Fairbank and collaborators have really discovered objects carrying fractional electric charge. I do not mean to endorse either result. I merely ask, *if* both experiments are right, how can we explain the apparent violation of the Dirac consistency condition? This question leads us into some desperate speculations.

2. The Dirac Quantization Condition

First, let us recall Dirac's[6] reasoning, which indicates that the results of Fairbank and Cabrera are incompatible. (For a more detailed review, see Ref. 12.)

Dirac envisaged a magnetic monopole as a semi-infinitely long, infinitesimally thin solenoid. One end of the solenoid, viewed in isolation, appears to be a magnetic charge. But it makes sense to identify this object as a magnetic monopole only if no conceivable experiment can detect the solenoid, in the limit in which it is infinitesimally thin.

We might imagine trying to detect the solenoid by doing an electron interference experiment; such an experiment gives a null result only if the phase picked up by the electron wave function, when the electron is transported along a closed path enclosing the solenoid, is trivial. Suppose a monopole with magnetic charge g sits at the origin, so that the magnetic field is

$$\vec{B} = g\,\frac{\hat{r}}{r^2}\,, \tag{1}$$

and that the solenoid lies on the negative z-axis. Then the vector potential can be written in polar coordinates as

$$\vec{A}\cdot d\vec{r} = g(1-\cos\theta)\,d\phi\,. \tag{2}$$

The electron interference experiment fails to detect the solenoid if

$$\text{phase} = \exp[-ie\oint\vec{A}\cdot d\vec{r}] = \exp[-4\pi i e g] = 1\,, \tag{3}$$

where $-e$ is the electron charge. Hence, we require the magnetic charge g to satisfy the (Dirac) quantization condition

$$g = n/2e\,, \tag{4}$$

where n is an integer. The minimal allowed charge $g_D = 1/2e$ is called the Dirac magnetic charge.

It may disturb you that, to derive the Dirac quantization condition, Eq. (4), I have used the electron charge $-e$. We would like to believe that quarks exist, and the electric charge of a down quark, for example, is $-e/3$. Will not the same argument as before, applied to a down quark instead of an electron, lead to the conclusion that the minimal allowed magnetic charge is $3g_D$ instead of g_D?

No, not if quarks are confined.[13] For if quarks are permanently confined in hadrons, it makes sense to speak of performing a quark interference experiment only over distances less than 10^{-13} cm, the size of a hadron. It is true that, when the down quark is transported around Dirac's solenoid, its wave function acquires the nontrivial phase

$$\exp[-i(e/3)\oint \vec{A}_{em}\cdot d\vec{r}] = e^{-i2\pi/3} \neq 1 \tag{5}$$

due to the coupling of the down quark to the electromagnetic vector potential, if the monopole carries the Dirac magnetic charge g_D. But we must recall that the down quark carries another degree of freedom, color. The solenoid is not detectable if the monopole also has a *color-magnetic field*, such that the phase acquired by the down quark wave function due to the color vector potential compensates for the phase due to the electromagnetic vector potential, or

$$\exp[ie_c\oint \vec{A}_{color}\cdot d\vec{r}] = e^{i2\pi/3} , \tag{6}$$

where e_c is the color gauge coupling.

The correct conclusion, then, if quarks are confined, is not that the minimal magnetic charge is $3g_D$, but rather that the monopole carrying magnetic charge g_D must also carry a color-magnetic charge. The color-

magnetic field of the monopole becomes screened by nonperturbative strong-interaction effects at distances greater than 10^{-13} cm.

How do we know that the color-magnetic field of the monopole is "screened"? Because there are no physical massless particles in QCD, the color-magnetic field can have no long-range effects. (In this respect, QCD differs from QED; long-range effects *are* mediated by the massless photon.) Either the color-magnetic field decays exponentially with distance r like $\exp(-Mr)$, where M is a hadron mass, or else the color-magnetic field is confined to flux tubes with a width of order $M^{-1} \sim 10^{-13}$ cm. But 't Hooft[14] has persuasively argued that color-magnetic flux tubes cannot form if quarks are exactly confined. The remaining alternative is that the color-magnetic field is screened.

Now we can state precisely the manner in which the observation of fractional electric charge is inconsistent with the observation of the Dirac magnetic charge. Suppose that there exists an *isolated* particle with electric charge $q = e/3$. By "isolated", I mean that it is possible to separate this particle from other charged particles by distances large compared to the color-magnetic screening distance M^{-1}. It *is* legitimate to apply Dirac's argument to *this* particle, and we conclude that the minimal allowed magnetic charge of a magnetic monopole is $3g_D$ rather than g_D. It is for this reason that the results of the Fairbank experiment and the Cabrera experiment appear to be incompatible.

I now wish to reformulate the above discussion of the color-magnetic field of the monopole using a different notation which will prove useful in the ensuing discussion. The vector potential of a magnetic monopole which, like that considered above, carries more than one type of magnetic charge, can in general be written[15]

$$\sum_a e_a \underset{\sim}{T}^a \vec{A}^a \cdot d\vec{r} = \tfrac{1}{2} \underset{\sim}{M} (1 - \cos\theta)\, d\phi \ , \qquad (7)$$

where $\underset{\sim}{M}$ is a constant matrix. The sum on a on the left-hand side of Eq. (7) runs over all unbroken gauge generators — for example, the electric charge and the eight color generators. The gauge couplings e_a have been absorbed into $\underset{\sim}{M}$. By an argument similar to that invoked above, we can derive the generalized Dirac quantization condition

$$\exp(2\pi i \underset{\sim}{M}) = \mathbb{1} \ . \qquad (8)$$

That is, $\underset{\sim}{M}$ must have integer eigenvalues.

For example, in the SU(5) grand unified model, the electric charge generator may be written as a 5×5 matrix

$$\underset{\sim}{Q}_{em} = \mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1) \ , \qquad (9)$$

where the diag(.,...,.) notation denotes a diagonal matrix with the indicated eigenvalues. The eigenvalues of $\underset{\sim}{Q}_{em}$ are the electric charges, in units of e, of the elements of the defining $\underset{\sim}{5}$ representation of SU(5) — antidown quarks in three colors, the neutrino, and the electron. The $SU(3)_c$ color generators are traceless 3×3 matrices acting only on the quarks; one of these is

$$\underset{\sim}{Q}_{color} = \mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0,0) \ . \qquad (10)$$

With the electric charge generator $\underset{\sim}{Q}_{em}$ given by Eq. (9), it is clear that objects which carry trivial $SU(3)_c$ triality have integer electric charge (in units of e), even though objects with nontrivial triality have fractional charge. A more mathematical, but useful, restatement of this observation is that $\exp(2\pi i \underset{\sim}{Q}_{em})$ is a nontrivial element of Z_3, the center of $SU(3)_c$; that is, it acts trivially on color singlets, and multiplies quarks of all three colors by the same nontrivial phase. For a monopole

which satisfies the quantization condition (8), the matrix M may be written as

$$\underset{\sim}{M} = n \underset{\sim}{Q}_{em} + m \underset{\sim}{Q}_{color} \ , \qquad (11)$$

where

$$\exp[2\pi i n \underset{\sim}{Q}_{em}] = \exp[-2\pi i m \underset{\sim}{Q}_{color}] = \underset{\sim}{Z} \ , \qquad (12)$$

and $\underset{\sim}{Z}$ is an element of Z_3; both n and m must be integers. The magnetic charge of this monopole is the coefficient of $e\underset{\sim}{Q}_{em}$ in $\tfrac{1}{2}\underset{\sim}{M}$, which is $n/2e = ng_D$.

The color-magnetic charge of the monopole must be defined with some care. The integer n is a conserved magnetic charge, but m is not, because the $U(1)_{em}$ and $SU(3)_c$ gauge groups have different topological properties. Color-magnetic monopoles with different values of m which correspond to the same element of Z_3, that is, values of m which are congruent modulo 3, are topologically equivalent to one another. Therefore, while the $U(1)_{em}$ charge can be any integer multiple of g_D, the color-magnetic charge may assume only three distinct values, which correspond to the three distinct elements of Z_3. We say that the monopole carries a conserved Z_3 color-magnetic charge. An assembly of three identical monopoles, each with a non-trivial Z_3 charge, has a trivial Z_3 charge. (How the conserved Z_3 flux can become screened is elucidated in Ref. 12.)

Two examples of monopoles in the SU(5) model are those defined by

$$\underset{\sim}{M} = \underset{\sim}{Q}_{em} + \underset{\sim}{Q}_{color} = \mathrm{diag}(0,0,1,0,-1) \ , \qquad (13)$$

and

$$\underset{\sim}{M} = 3\underset{\sim}{Q}_{em} = \mathrm{diag}(1,1,1,0,-3) \ ; \qquad (14)$$

both are consistent with the quantization condition, Eq. (8). The monopole defined by Eq. (13) has $U(1)_{em}$ magnetic charge g_D and a nontrivial Z_3 color-magnetic charge; the monopole defined by Eq. (14) has $U(1)_{em}$ magnetic charge $3g_D$ and trivial Z_3 color-magnetic charge. In the SU(5) model, the lightest monopole, and therefore the stable one, is the one defined by Eq. (13).

3. Interpretations of Fractional Electric Charge

The above argument seems to exclude, on the basis of very general considerations, the possibility that both monopoles with magnetic charge $g_D = 1/2e$ and isolated fractional charges with $q = e/3$ exist. We will now hold this argument up to further scrutiny, discussing three different theoretical interpretations of the observation of fractional electric charge.

(a) Unconfined Quarks

The interpretation of the observation of electric charge e/3 which seems to require minimal speculation is that quarks are not exactly confined, and Fairbank has detected free quarks.[16] But we have concluded that if any isolated charge -e/3 particles exist, then the minimal allowed magnetic charge of a monopole is $3g_D$, contrary to Cabrera's observation.

Further insight into this conclusion is gained if we recall another result obtained by 't Hooft.[14] I noted earlier that 't Hooft argued that if quarks are exactly confined, then the conserved Z_3 color-magnetic charge is screened rather than confined. The converse is also true! If quarks are not exactly confined, that is, if physical states exist which carry non-trivial color triality, then the Z_3 color-magnetic charge *is* confined, assuming that there are no long-range color forces (see below).

Consider, for example, how the monopoles of the SU(5) grand unified model would be affected if the $SU(3)_c$ gluons acquired masses of order μ due to the Higgs mechanism.[1] The stable SU(5) monopoles carry $U(1)_{em}$ magnetic charge g_D and nontrivial Z_3 color-magnetic charge. The Z_3 color-magnetic flux emerging from these monopoles would collapse to a tube with a width of order μ^{-1}, and the monopoles would be confined to magnetically neutral "mesons" or to "baryons" with trivial Z_3 color-magnetic charge and $U(1)_{em}$ magnetic charge $3g_D$. The magnetic baryons would thus carry the minimal

magnetic charge allowed by the Dirac quantization condition, in the presence of free quarks.

The above remarks require one qualification. The existence of a free quark is consistent with the existence of a free monopole with $U(1)_{em}$ magnetic charge g_D if the monopole carries a *longe-range* Z_3 color-magnetic field; that is, if the color-magnetic field is not screened. This theoretical possibility must be excluded by appealing to experiment. There can be long-range color forces only if there are massless hadrons. If there were massless hadrons, then they would be copiously produced in hadronic collisions, and we would surely know about it.

To summarize: If there are no long-range color interactions, then either quark confinement is exact, in which case we cannot interpret Fairbank's fractional charge as a free quark, or else monopoles with magnetic charge g_D are confined, in which case Cabrera could not have detected an isolated monopole with that charge. (We can safely rule out the possibility that the monopole confinement scale μ^{-1} is macroscopic, for in that case the probability of finding any free quarks would be negligible.) On the other hand, there can be long-range color interactions only if there are massless hadrons, which is experimentally excluded. To reconcile the observations of Fairbank and Cabrera, we must find a different interpretation of the observed fractional charge.

(b) Dyons

One amusing possibility is that the objects which carry fractional electric charge are actually "dyons" which also carry magnetic charge. For two dyons with electric and magnetic charges (q_1, g_1) and (q_2, g_2), the Dirac consistency condition becomes generalized to[17]

$$q_1 g_2 - q_2 g_1 = n/2 \qquad (15)$$

where n is an integer. This condition is satisfied if the electric charge q of an object with magnetic charge g is given by

$$q/e = m + e\,g\,\theta/\pi \tag{16}$$

where m is an integer, and θ, which may take any value from $-\pi$ to π, is a CP-violating parameter of electrodynamics which has observable effects only in the presence of a monopole.[18] The minimal electric charge of a dyon with the Dirac magnetic charge is $q = e\theta/2\pi$. Conceivably, Fairbank and Cabrera have discovered the same object — a dyon; Fairbank has measured its electric charge and Cabrera has measured its magnetic charge.

Is there any reason to expect $|\theta|/2\pi \sim 1/3$? I know of no such reason. On the other hand, $|\theta|/2\pi \sim 1/3$ cannot be excluded. In grand unified theories, the θ appearing in Eq. (16) is related to another angle,[19] θ_{QCD}, which *is* known to be very small (less than 10^{-9}).[20] But a natural explanation for the small value of θ_{QCD}, the Peccei-Quinn mechanism,[21] has been proposed, and this mechanism permits θ to assume a value which need not be small in general.

In any event, this theoretical possibility appears to be experimentally excluded. Cabrera[22] has measured the magnetic charges of two niobium spheres on which fractional electric charge had been observed; the measured magnetic charges were consistent with zero.

(c) <u>Fractionally Charged Leptons</u>

The remaining possibility is that Fairbank has observed a color-singlet particle with vanishing magnetic charge and fractional electric charge. It could be either a fractionally charged lepton or a hadron containing a colored particle with an exotic charge.

How can we reconcile the existence of such a particle with Cabrera's

observation of a monopole with magnetic charge g_D? Our discussion of the generalized quantization condition, Eq. (8), suggests an answer.[7-11] There must be a new long-range gauge interaction, not yet observed, in addition to electromagnetism. This new interaction might be a nonabelian gauge interaction with a macroscopic confinement length,[24] but I will consider the simplest possibility, that it is a U(1) gauge interaction. I will refer to this interaction as extraordinary electromagnetism, and to its gauge group as $U(1)_{ex}$; matter carrying $U(1)_{ex}$ charge will be called extraordinary matter.

Extraordinary electromagnetism may have escaped detection so far because ordinary matter is neutral under $U(1)_{ex}$. But Fairbank's fractionally charged particle and Cabrera's monopole must carry both types of charge. The Dirac quantization condition for an electrically charged particle with $U(1)_{em}$ and $U(1)_{ex}$ charges q and q' and a monopole with magnetic charges g and g' is

$$q\,g + q'g' = n/2\ . \tag{17}$$

This condition is satisfied by, for example, a monopole with magnetic charges $g = g_D = 1/2e$ and $g' = g_D' = 1/2e'$, and an extraordinary lepton with electric charges $q = e/3$ and $q' = -e'/3$. The apparent inconsistency with the Dirac quantization condition may have arisen, then, because Fairbank and Cabrera each measured only one of the two (or more) types of charge carried by the lepton and monopole.

We should next consider whether we can construct realistic grand unified models which take advantage of this possibility.

4. Extraordinary Model Building

Howard Georgi and I have made some preliminary attempts to construct such models,[25] as have others.[9-11] We seek a model in which the low-energy gauge group

$$G_{LE} = SU(3)_c \times [SU(2) \times U(1)]_{ew} \times U(1)_{ex} \quad (18)$$

is embedded in some unifying group G. The breakdown of G to G_{LE} might occur all at once at the grand unification mass scale, or there might be a nontrivial sequence of intermediate symmetry-breaking scales. We require that there are at least three generations of ordinary matter, which are neutral under $U(1)_{ex}$ and carry the standard $SU(3)_c \times [SU(2) \times U(1)]_{ew}$ quantum numbers of

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^{1/6}, \quad \bar{u}_L^{-2/3}, \quad \bar{d}_L^{1/3}, \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L^{-1/2}, \quad \bar{e}_L^{1} . \quad (19)$$

There must also be a magnetic monopole with $U(1)_{em}$ magnetic charge g_D = 1/2e, and a color-singlet charged particle with $U(1)_{em}$ electric charge e/3.

The smallest unifying group for which a model satisfying these criteria is likely to be found is G = SU(7), and the most promising fermion representation to consider in SU(7) is the representation

$$\underset{\sim}{7} + \underset{\sim}{\overline{21}} + \underset{\sim}{35} , \quad (20)$$

which is obtained by decomposing the spinor representation of SO(14) with respect to SU(7). To begin our search, let us ask whether there is any embedding of G_{LE} in SU(7) such that the representation (20) contains at least one generation of ordinary matter, neutral under $U(1)_{ex}$. This is a variation on a game pioneered by Kim,[26] who investigated the consequences of different embeddings of $SU(3)_c \times [SU(2) \times U(1)]_{ew}$ in SU(7). Kim's motivation was to understand the repeated generation pattern of the quarks and leptons, and we might hope that models which satisfy our criteria will also offer some insight into the generation puzzle.

It turns out that there are only two solutions to the problem of embedding G_{LE} in SU(7) such that the representation (20) contains at least one ordinary generation with conventional quantum numbers. In both solutions, $SU(3)_c \times SU(2)_W$ is embedded in the standard way: 7 → (3,1) + (1,2) + 2(1,1). The generators of $U(1)_{em} \times U(1)_{ex}$, in the first solution, are

$$\underset{\sim}{Q}_{em} = \mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1,0,0)$$

$$\underset{\sim}{Q}_{ex} = \mathrm{diag}(0,0,0,0,0,\tfrac{1}{2},-\tfrac{1}{2})$$

where

$$\underset{\sim}{Q}_{color} = \mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0,0,0,0) . \quad (21)$$

This is the trivial solution. The Georgi-Glashow SU(5) has been embedded in SU(7), and $U(1)_{ex}$ commutes with this SU(5). This model does not satisfy our criteria, because all particles with trivial color triality carry integer electric charge. Nevertheless, it is not totally uninteresting from the perspective of the generation puzzle. The representation (20) contains two ordinary left-handed SU(5) generations, neutral under $U(1)_{ex}$, plus two right-handed generations which carry $U(1)_{ex}$ charges. The unbroken $U(1)_{ex}$ gauge symmetry prevents the left-handed and right-handed generations from pairing up and acquiring superheavy masses. Including an unbroken $U(1)_{ex}$ thus allows us to unify two ordinary SU(5) generations in the spinor representation of SO(14). This unification of two generations is similar to that achieved by Kim,[26] who considered unconventional embeddings of $U(1)_{em}$ in SU(7).

The other, nontrivial, solution is:

$$\underset{\sim}{Q}_{em} = \mathrm{diag}(\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},0,-1,\tfrac{1}{2},0) ,$$

$$\underset{\sim}{Q}_{ex} = \mathrm{diag}(\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},0,0,-\tfrac{1}{2},-1) ,$$

$$\underset{\sim}{Q}_{color} = \mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},-\tfrac{2}{3},0,0,0,0) . \qquad (22)$$

Because $\underset{\sim}{Q}_{em} \pm \underset{\sim}{Q}_{ex} + \underset{\sim}{Q}_{color}$ has integer eigenvalues, this model has magnetic monopoles with $U(1)_{em}$ magnetic charge $g = g_D = 1/2e$ and $U(1)_{ex}$ magnetic charge $g' = g'_D = \pm 1/2e'$. However it is also clear that, with this embedding of $\underset{\sim}{Q}_{em}$, objects with trivial $SU(3)_c$ triality carry *half*-integer electric charge, instead of one-third integer charge.

A surprising feature of this model is that the Weinberg angle comes out right; the unrenormalized value of $\sin^2\theta_W$ is 3/8 as in the SU(5) model. But, because it contains half-integer charges rather than one-third integer charges, this model cannot account for the Fairbank experiment. No SU(7) model can, if the fermions are in the representation (20), and the model also contains magnetic monopoles with magnetic charge g_D.

What should we try next? We could consider other representations of SU(7), semisimple unifying groups like SU(5) × SU(5), or simple groups larger than SU(7). It is not hard to construct models based on SU(5) × SU(5) which satisfy our criteria,[9] but I will not discuss this construction here. Instead, I will describe two models based on SU(9).

Both of these models satisfy all our criteria. In the first model, $SU(3)_c$ and $SU(2)_W$ are minimally embedded in SU(9), and the generators of $U(1)_{em} \times U(1)_{ex}$ are

$$\underset{\sim}{Q}_{em} = \mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1,-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0) ,$$

$$\underset{\sim}{Q}_{ex} = \mathrm{diag}(0,0,0,0,0,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},-1) ,$$

where

$$\underset{\sim}{Q}_{color} = \mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0,0,0,0,0,0) . \qquad (23)$$

(This model was also proposed in Ref. 10.) In the second model, the embedding of $SU(3)_c$ in SU(9) is nonstandard; we have $9 \to 3+3+1+1+1$, and the generators of $U(1)_{em} \times U(1)_{ex}$ are

$$\underset{\sim}{Q}_{em} = \mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,-1,0,0,0,0) ,$$

$$\underset{\sim}{Q}_{ex} = \mathrm{diag}(0,0,0,0,0,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},-1) ,$$

where

$$\underset{\sim}{Q}_{color} = \mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0,0,-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0) . \qquad (24)$$

In both models, $\underset{\sim}{Q}_{em} + \underset{\sim}{Q}_{ex} + \underset{\sim}{Q}_{color}$ has integer eigenvalues, and both models therefore contain magnetic monopoles with magnetic charges g and g' given by

$$(e\,g, e'g') = (\tfrac{1}{2},\tfrac{1}{2}) . \qquad (25)$$

Also, in both models, objects with trivial color triality need not have integer electric charge; instead, it is required that

$$q/e + q'/e' = n \qquad (26)$$

where n is an integer and q/e may be one third of an integer.

The fermions may be chosen to lie in the SU(9) representation

$$\underset{\sim}{9} + \overline{\underset{\sim\sim}{36}} + \underset{\sim\sim}{84} + \overline{\underset{\sim\sim\sim}{126}} \qquad (27)$$

which is obtained by decomposing the spinor representation of SU(18) with respect to SU(9). This representation contains, in both models, two ordinary generations which are neutral under $U(1)_{ex}$, plus many extraordinary fermions. For example, in the second model, with embedding (24), there are extraordinary leptons with charges

$$(q/e, q'/e') = (-\tfrac{1}{3},\tfrac{1}{3}),(-\tfrac{1}{3},-\tfrac{2}{3}),(\tfrac{1}{3},-\tfrac{1}{3}),(\tfrac{1}{3},\tfrac{2}{3}),(\tfrac{2}{3},\tfrac{1}{3}),(\tfrac{2}{3},-\tfrac{2}{3}),(-\tfrac{2}{3},-\tfrac{1}{3}),(-\tfrac{2}{3},\tfrac{2}{3}). \qquad (28)$$

The model with the embedding (24) appears to have the advantage that the unrenormalized Weinberg angle at the unification scale is $\sin^2\theta_W = 3/8$, as in the SU(5) model. However, because of the nonstandard embedding of $SU(3)_c$, $\sin^2\theta_W$ is renormalized differently, and it is actually necessary in both models to introduce intermediate symmetry-breaking scales in order to get the observed value of $\sin^2\theta_W$ at low energy. This is a heavy price to pay, because our power to compute $\sin^2\theta_W$ is lost. The renormalized value at low energy of the extraordinary gauge coupling e' also depends on the intermediate-scale physics, but it is typically of order e. The Higgs structure needed to achieve the desired symmetry-breaking pattern is quite intricate; I will not describe it.

5. Extraordinary Phenomenology

In all models of the type described in Section 4, there must be at least one stable particle which carries fractional $U(1)_{em}$ and $U(1)_{ex}$ charge. Because this particle has not been produced at Petra, we know that its mass m_{ex} satisfies

$$m_{ex} \geqslant 20 \text{ GeV} . \qquad (29)$$

Because its mass is forbidden by weak-interaction symmetries, we also know that it cannot be arbitrarily heavy; we expect

$$m_{ex} \leqslant G_F^{-\frac{1}{2}} \sim 300 \text{ GeV} . \qquad (30)$$

Apparently, extraordinary quarks and leptons are much heavier than their ordinary counterparts. I do not know why this should be so, but perhaps the question should be phrased differently. The natural value for a mass forbidden by weak-interaction symmetries is of order 300 GeV; what is truly mysterious is not that the extraordinary matter is so heavy, but that the ordinary matter is so light.

If ordinary quarks and leptons are neutral under $U(1)_{ex}$, as we have assumed, then the $U(1)_{ex}$ extraordinary photon couples to ordinary matter only through processes involving virtual extraordinary matter. For example, the extraordinary photon can couple to three gluons through an extraordinary quark loop, but, because of gauge invariance, this coupling is suppressed at low energy by m_{ex}^{-4}. There is no direct experimental evidence, or any astrophysical evidence, against the existence of a massless particle which couples so weakly to ordinary matter.

However, models of this type face a serious difficulty when we consider their cosmological implications. The difficulty, which is encountered by any model which contains fractionally charged color-singlet particles, is that too many such particles are produced in the early universe. Standard estimates indicate that, when the electroweak and extraordinary interactions freeze out and annihilation of extraordinary matter ceases, the relative abundance of extraordinary matter to baryons is given by

$$n_{ex}/n_{baryon} \gtrsim 10^{-5}(m_{ex}/100\ \text{GeV}) . \qquad (31)$$

Observational bounds on the abundance of fractionally charged particles in the earth's crust are highly uncertain, but it is generally agreed that the abundance must be much smaller than allowed by (31), for $m_{ex} \gtrsim 20$ GeV.

A means of reducing an initially large abundance of fractionally charged leptons to an acceptable level has been proposed by Goldberg.[27] He suggests that most of the leptons annihilated in very massive first-generation stars. The problem with this suggestion is that, to explain the low abundance of fractionally charged particles in the earth's crust, it is required that all but a tiny fraction of the matter in the solar system has been processed in these first-generation stars, which seems implausible.

A more reasonable way to avoid a large cosmological abundance of fractionally charged particles is to construct a model in which all fractionally charged particles are superheavy. In the new inflationary universe scenario,[28] it is possible for the abundance of stable superheavy particles to be both non-negligible and small. Indeed, this scenario provides the best known way to obtain an acceptable cosmological abundance of magnetic monopoles (for a review, see Ref. 7); we might as well use the same trick to control the abundance of fractionally charged particles.

It is possible to construct models in which all extraordinary matter is superheavy, but not without paying an aesthetic price. Barr, Reiss, and Zee[9] recently constructed such a model based on $SU(5)_{ord} \times SU(5)_{ex}$, in which

ordinary matter carries $SU(5)_{ord}$ quantum numbers and extraordinary matter carries $SU(5)_{ex}$ quantum numbers. The two SU(5)'s are treated quite asymmetrically, so that in a sense this model is less unified than the SU(9) model described earlier. But it is nonetheless a quite simple model which satisfies all the criteria specified at the beginning of Section 4, and avoids the cosmological problem as well.

6. Conclusions

The observation of a magnetic monopole carrying the Dirac magnetic charge $g_D = 1/2e$ and the observation of isolated electric charge $q = e/3$ on matter are not necessarily incompatible, but the two results together impose severe constraints on unified model building. Two types of models have been described which can accommodate both observations. One type of model predicts the existence of many new particles with mass less than a few hundred GeV which will turn up in forthcoming accelerator experiments, but runs into cosmological problems. The other type of model avoids the cosmological problems by making all fractionally charged particles superheavy; the new physics predicted by this type of model will not be directly accessible in accelerator experiments.

The model-building exercises outlined here were clearly very *ad hoc*. Our goal was to construct models which are consistent with the results of both the Cabrera experiment and the Fairbank experiment, and in this we succeeded. But one might have hoped that these models would have other, unexpected, good features; for example, one might have hoped to gain insight into the generation puzzle. Unfortunately, this hope was not realized.

Nevertheless, it is probably wise to bear in mind the implications of the Cabrera experiment and the Fairbank experiment taken together, which are much more far-reaching than the implications of either experiment taken alone. These implications provide us with further motivation to follow with great interest the developing experimental status of fractional charge and magnetic monopoles.

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