CP Nonconservation without Elementary Scalar Fields

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Dynamically broken gauge theories of electroweak interactions provide a natural mechanism for generating CP nonconservation. Even if all vacuum angles are unobservable, strong CP nonconservation is not automatically avoided. In the absence of strong CP nonconservation, the neutron electric dipole moment is expected to be of order \(10^{-24}\) e \(\cdot\) cm.

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In this Letter, we show that there is a natural mechanism for generating CP nonconservation in dynamically broken electroweak gauge theories. Our proposal requires no new gauge interactions beyond those discussed previously.\(^1\) \(^\text{\textsuperscript{1-4}}\) Spontaneous CP nonconservation can appear when we carry out Dashen's procedure to identify the correct chiral vacuum. Whether CP nonconservation occurs is determined in principle by only the gauge group and the fermion representation content of the theory.

These theories have no elementary scalar fields and no fermion bare masses. Even though there are no observable vacuum angles, strong CP nonconservation\(^1\) \(^\text{\textsuperscript{1-4}}\) is not automatically avoided.\(^5\) We state a criterion for the absence of strong CP nonconservation; if it is satisfied, CP-nonconserving phases in the quark mass matrix are naturally suppressed by a factor of order \(10^{-6}\). Additional CP nonconservation appears in the electroweak interaction and in the gauge interaction responsible for chiral symmetry breaking. We predict that the electric dipole moment of the neutron is of order \(10^{-24}\) e \(\cdot\) cm.

In a theory of weak interactions without elementary scalar fields, a new gauge interaction\(^1\) \(^\text{\textsuperscript{1-4}}\) ("hypercold," with gauge group \(G_H\)) is required, in addition to the familiar color \([G_C = SU(3)]\) and electroweak \([G_W = SU(2) \otimes U(1)]\) interactions. Hypercolor becomes a strong interaction at mass scale \(m_H \approx 1\) TeV, and drives the breaking of \(G_W\) down to \(U(1)_{\text{EM}}\).

A theory with gauge group \(G_H \otimes G_C \otimes G_W\) alone cannot be realistic. A "sideways" interaction\(^3\) \(^\text{\textsuperscript{3-4}}\) (with group \(G_E\)) is needed to break explicitly all chiral symmetries not gauged by \(G_W\). \(G_S\) is dynamically broken to a subgroup containing \(G_H \otimes G_C\) at a mass scale \(m_S \approx 100\) TeV. (We need not speculate here on the origin of the \(G_S\) breakdown.)\(^3\) \(^\text{\textsuperscript{3-10}}\)

All fermions are in at most four irreducible representations of \(G_S\).\(^3\) In the effective gauge theory which describes physics below 100 TeV, each of these representations transforms reducibly under \(G_H \otimes G_C\). If we neglect the broken sideways interactions and the weak interactions, the \(G_H \otimes G_C\)-invariant effective Hamiltonian \(H_0\) respects a global (chiral) flavor-symmetry group \(G_F\). \(G_W\) is a subgroup of \(G_F\).

When hypercolor and color become strong, \(G_F\) is dynamically broken to a subgroup \(S_F\). Many Goldstone bosons result. Three of these are absorbed by the weak \(W^\pm\) and \(Z^0\) bosons.\(^1\) \(^\text{\textsuperscript{1-2}}\) The remaining Goldstone bosons acquire mass from the chiral-symmetry-breaking perturbation \(\mathcal{V}_C\) generated by the weak and sideways interactions.

The ground state of \(\mathcal{V}_C\) is highly degenerate; the vacua are parametrized by the coset space...
$G_f/S_f$. The perturbation $\mathcal{K}'$ lifts the degeneracy and picks out the true chiral-perturbative vacuum, the limit of the ground state of $\mathcal{K}_{\mu} + \epsilon \mathcal{K}'$ as $\epsilon \to 0$.

AsDashen\textsuperscript{5} explained, we can identify the correct vacuum by minimizing an effective potential which, in lowest-order perturbation theory, is
\begin{equation}
V(g) = \langle \Omega | U^{-1}(g) \mathcal{K}' U(g) | \Omega \rangle.
\end{equation}

Here $g \in G_f$, $U(g)$ represents $G_f$ in the Hilbert space of states, and $| \Omega \rangle$ is the $S_f$-invariant vacuum. In the calculations described below, it is convenient to regard the symmetry group of the vacuum as fixed. Then, from among all $G_f$-equivalent perturbation $\mathcal{K}'(g) = U^{-1}(g) \mathcal{K}' U(g)$, we choose that one which minimizes the energy of the $S_f$-invariant vacuum $| \Omega \rangle$.

Now we see how spontaneous CP nonconservation can arise naturally. We assume that the fermion representation content under $G_S$ vacuum angles\textsuperscript{6} can be simultaneously rotated to zero, and hence are unobservable. In particular, $G_S$ might be simple.\textsuperscript{11} We further assume that the breaking of $G_S$ does not introduce CP nonconservation, so that the effective gauge theory below 100 TeV is CP invariant.\textsuperscript{12,13} Thus the vacuum $| \Omega \rangle$ and the perturbation $\mathcal{K}'$ are CP invariant, and the effective potential is CP symmetric. However, the energy might be minimized by a CP-nonconserving $\mathcal{K}'(g)$; then $V(g)$ has a degenerate minimum, and CP is spontaneously broken.\textsuperscript{14} The minimum of $V(g)$, and whether spontaneous CP nonconservation occurs, are determined once the pattern of $G_S$ and $G_f$ breaking is known.

The approximate flavor group $G_f$ is determined by the $G_H \otimes G_C$ representation content of the fermions. We will assume that all nontrivial representations of $G_H \otimes G_C$ are complex, that the color and maximal hypercolor interactions are vectorial, and that the maximal isospin is left unbroken when hypercolor and color get strong. [These assumptions ensure that $\chi_H$ breaks down to $U(1)_{EM}$ and that the relation $M_y/M_Z = \cos \theta_W$ is satisfied.\textsuperscript{2}]

Then the fermions may be denoted $\psi_{Lr}(\rho), \psi_{Rr}(\rho)$. The index $\rho$ identifies the irreducible representation $\mathcal{X}(\rho)$ according to which $\psi(\rho)$ transforms under $G_H \otimes G_C$. The gauge group $G_H \otimes G_C$ acts on the index $i$. The index $r (r = 1, 2, \ldots, n_\rho)$ labels the various flavors of fermions which transform as the representation $\mathcal{X}(\rho)$.

The flavor group is
\begin{equation}
G_f = \prod_\rho [SU(n_\rho) \otimes SU(n_\rho) \otimes U(1)^*(1)] \otimes U_A(1)^* S_f,
\end{equation}

which breaks down to $S_f = \prod_\rho [SU(n_\rho) \otimes U(1)^*(1)]$.

Under $G_f$, the fermions transform as
\begin{equation}
\psi_{Lr}(\rho) \rightarrow W_{rr'} L(\rho) \psi_{Lr'}(\rho),
\end{equation}
\begin{equation}
\psi_{Rr}(\rho) \rightarrow W_{rr'} R(\rho) \psi_{Rr'}(\rho),
\end{equation}

where $W_{rr'}(\rho)$ and $W_{rr'}(\rho)$ are unitary $n_\rho \times n_\rho$ matrices.

If $S_f$ is the diagonal subgroup with $W_{rr'} = \delta_{rr'}$, then the elements of $G_f/S_f$ can be labeled by a set of unitary matrices $W^{\pm}(\rho) = W^{\pm}(\rho)^* W^{\pm}(\rho)$. Each representation $\mathcal{X}(\rho)$ of $G_H \otimes G_C$ has a hypercolor and color anomaly given by
\begin{equation}
\delta \rho \cdot J_{\rho}^H(\rho) = T_{\rho}^H \left( \frac{\mu_\rho^2}{8 \pi^2} \right) tr F_H F_H^c + T_{\rho}^C \left( \frac{\mu_\rho^2}{8 \pi^2} \right) tr F_C F_C^c,
\end{equation}

where $T_{\rho}^H (T_{\rho}^C)$ is the trace of the color (hypercolor) squared generators in the representation $\mathcal{X}(\rho)$. (We have assumed that $G_H$ is simple.) Only those linear combinations of the $J_{\rho}^H(\rho)$'s with vanishing hypercolor and color anomalies generate $U_A(1)$ symmetries which are included in $G_f$. Thus the $W^{\pm}(\rho)$'s satisfy the constraint
\begin{equation}
\prod_\rho [\det W(\rho)] T_{\rho}^{H} = \prod_\rho [\det W(\rho)] T_{\rho}^{C} = 1.
\end{equation}

The weak interactions play a secondary role in determining the minimum of the effective potential\textsuperscript{15}; we need consider only the broken sideways interactions. We integrate out the massive sideways gauge bosons to obtain a $G_H \otimes G_C \otimes G_m$-invariant effective Lagrangian. The leading $G_f$-breaking terms in the effective Lagrangian are four-fermion operators. Higher-dimension operators are suppressed by additional powers of $m_\nu^{-1}$. We simplify the discussion by assuming that $SU(2)_W$ commutes with $G_S$, so that there is an exact global $U(1)$ symmetry which distinguishes weak doublets from weak singlets. After Fierz rearrangements, the most general four-fermion operator invariant under $G_H \otimes G_C \otimes G_m$ and the global $U(1)$ which contributes to the effective potential has the form\textsuperscript{16}
\begin{equation}
3 \mathcal{L}' = \prod_{i, j, k, l} \gamma_{ij, kl}^{\mu_\rho, \nu_\sigma} \psi_{Lr}(\rho) \psi_{Rr}(\rho) \gamma_{ijkl}^{\mu_\rho, \nu_\sigma} \psi_{Le}^{(\rho)} \psi_{Re}^{(\rho)}.
\end{equation}

Here the $\gamma_{ij, kl}^{\mu_\rho, \nu_\sigma}$ are $G_H \otimes G_C$-invariant tensors, indexed by $\mu_\rho$ and $\nu_\sigma$, and $\Gamma_{ij, kl}^{\mu_\rho, \nu_\sigma} = O(m_\nu^{-2})$ is a $G_m$-invariant tensor which can be calculated to arbitrary order in the sideways coupling $g_\delta^2$. To lowest order in $g_\delta^2$, we have $\mathcal{L}' = g_\delta^2 \mu_\sigma \omega^{ij, kl} \mu_\rho \nu_\sigma \psi_{Le}^{(\rho)} \psi_{Re}^{(\rho)}$, where $\mu_\sigma$ is the massive sideways gauge boson mass matrix, and the $J_{\rho}(\mu_\sigma)$ are the left-(right-) handed broken sideways currents. This interaction can be Fierz transformed into the form of Eq. (6).

To compute the effective potential, we note that the $S_f$-invariant vacuum $| \Omega \rangle$ has the property
\[ \langle \Omega | \bar{\psi}_{LR_{\rho}} | \psi_{LR_{\rho}} \rangle \approx \delta_\rho | \bar{\psi}_{LR_{\rho}} \psi_{LR_{\rho}} \rangle. \]

Therefore, \( S_\rho \) invariance implies that
\[ \langle \Omega | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle = \delta_\rho | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle. \]

The second term on the right-hand side of Eq. (7) is \( G_s \) invariant. Hence, the leading term in the effective potential is
\[ V(W) = \Delta G_s \gamma_\mu | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \]

Hermiticity and CP invariance of the effective Hamiltonian require
\[ G_s \gamma_\mu | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle = \Delta G_s \gamma_\mu | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle. \]

CP invariance of the vacuum \( | \Omega \rangle \) implies \( \Delta G_s \gamma_\mu | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle = \Delta G_s \gamma_\mu | \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle. \) Therefore, \( V(W) = V(W^\ast) \). In general, we expect that for some range of sideways gauge-boson masses, the minimum of \( V(W) \) occurs for \( W \) complex, \( W \neq W^\ast \). Then the minimum is degenerate, and \( CP \) is spontaneously broken.

Now we minimize \( V(W) \) to determine the correct chiral perturbation \( \mathcal{S}(\mathcal{W}) \). \( V(W) \) is stationary under the constraint in Eq. (5) iff
\[ \left( M^{(\bar{\rho})} - M^{(\rho)} \right) \Delta \rho = \left( \mathcal{W} \right) \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right), \]

where
\[ M_{\rho \gamma, \rho \gamma} \Delta \rho = \left( \mathcal{W} \right) \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \]

Here \( \nu_H \) and \( \nu_C \) are Lagrange multipliers. [Taking the trace of both sides of Eq. (9) and summing on \( \rho \) we obtain a relation between \( \nu_H \) and \( \nu_C \). If \( G_s \) is simple, then \( \nu_H = - \nu_C \).]

If we include contributions from higher-dimension operators to \( V(W) \), the condition for an extremum of \( V(W) \) is still of the form given in Eq. (9), but with a modified matrix \( M^{(\bar{\rho})} \). For quarks, this matrix \( M^{(\bar{\rho})} \) differs from the "current-algebra" quark mass matrix by approximately one part in \( 10^7 \). This relation holds because the mass \( m_q \sim 300 \text{ MeV} \) at which color becomes strong is small compared to the mass \( m_q \sim 1 \text{ TeV} \) at which hypercolor becomes strong.

Physics below \( m_q \sim 1 \text{ TeV} \) can be described by an effective theory involving only quarks, leptons, gluons, electroweak bosons, and pseudo-Goldstone bosons. This effective Lagrangian is obtained by integrating out hypergluons and hyperfermions, as well as massive sideways bosons. We must also sum up all hard (order \( m_q \)) gluon exchanges. A series of \( G_s \)-invariant operators is generated. The operators which contribute to the effective potential are \( A_{\rho \gamma, \rho \gamma} \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \), \( B_{\rho \gamma, \rho \gamma} \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \), \( C_{\rho \gamma, \rho \gamma} \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \), and higher-dimensional operators. The coefficients \( A \), \( B \), and \( C \) depend on the hyperfermion \( W^{(\rho)} \)\( s \). A is the quark mass matrix; it contributes \( 2 \Delta G_s \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \) to \( V(W) \). Relative to this term, the contribution from four-quark operators is suppressed by \( (m_c/m_u)^2 \sim 10^{-10} \). The operator \( \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \) is generated by graphs involving one hard-gluon exchange\( ^{15} \); its contribution is suppressed by \( (\alpha_c m_q^2)/(2\pi) \sim 10^{-9} \). Hence, the part of \( V(W) \) involving \( \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \) is \( 2 \Delta G_s \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \)\( A[1 + (10^{-9})] \) and therefore \( M^{(\bar{\rho})} = \mathcal{W} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) A[1 + (10^{-9})] \).

Equation (9) is solved by several sets of unitary matrices \( W = \mathcal{W} (\rho) \). From all solutions, we choose the one (or ones) which minimizes \( V(W) \). The W which minimizes the effective potential will satisfy one of three conditions:

(i) \( W = W^\ast \). In this case, \( CP \) invariance is not spontaneously broken, and no \( CP \) nonconservation occurs at all.

(ii) \( W \neq W^\ast \), \( \nu_C \neq 0 \). In this case, strong \( CP \) nonconservation occurs. The effective Hamiltonian contains the \( CP \)-nonconserving term \( \frac{1}{\Delta} \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \)\( A \) a current-algebra calculation\( ^{15} \) of the neutron electric dipole moment \( D_n \) yields \( D_n \sim 2 \times 10^{-16} (m_c/m_u) e \cdot \text{cm} \), where \( m_u \sim 5 \text{ MeV} \) is the up quark mass. The natural scale of \( \nu_C/\Delta \) is of order \( m_u \) and so we expect \( D_n \sim 10^{-15} e \cdot \text{cm} \), which exceeds the experimental bound\( ^{15,21} \) by a factor of \( 10^8 \).

(iii) \( W \neq W^\ast \), \( \nu_C = 0 \). In this case, \( CP \) invariance is spontaneously broken, but there is no large contribution to \( D_n \). The coefficients \( B \) and \( C \) have phases of order \( 1 \), but since \( M^{(\bar{\rho})} \) is Hermitian, \( \mathcal{W} (\rho) \mathcal{A} \) is Hermitian up to corrections of order \( 10^{-9} \). The anti-\( \text{Hermitian} \) part of the quark mass matrix contributes of order \( 10^{-24} e \cdot \text{cm} \) to \( D_n \). A similar contribution comes from the operator \( \mathcal{W} (\rho) \mathcal{A} \)\( \mathcal{V} \left( \bar{\psi}_{LR_{\rho}} \gamma_\mu | \psi_{LR_{\rho}} \rangle \right) \), where \( F^{(\rho )} \) is the photon field. Its coefficient is expected to have a phase of order \( 1 \) and a magnitude \( (\alpha_c m_q^2)/(2\pi) \times (m_u/m_q)^2 e \sim 10^{-24} e \cdot \text{cm} ^{15,21} \). The Lagrange multiplier \( \nu_C \) vanishes if the minimum of the effective potential remains a stationary point when we remove the constraint that \( U_{\alpha}(1) \) rotations with a color anomaly are not allowed.

In any given model, the dynamics determines which possibility is realized. Only models of type (iii) can have \( CP \)-nonconserving interactions consistent with experiment.

In perturbation theory about the true chiral vacuum, the \( \mathcal{W} (\rho) \)\( s \) appear explicitly in the weak and sideways currents. Minimizing the effective potential determines \( \mathcal{W} (\rho) = \frac{1}{\Delta} \mathcal{V} (\rho) \mathcal{W} (\rho) \), but does not determine \( \mathcal{W} (\rho) \) and \( \mathcal{W} (\rho) \) separately. For
quarks, we define $W_\alpha^L$ and $W_\alpha^R$ by demanding that the mass matrix $W_\alpha$ be diagonal. The quark matrices $W_\alpha W_\alpha^T$ commute with electric charge; there are unitary matrices $W_{d_1}^{L,R}$, $W_{d_2}^{L,R}$ for up and down quarks. The Kobayashi-Maskawa mixing matrix is $W_{d_1}^{L} W_{d_2}^{L}$. If it contains phases which cannot be removed by redefining the phases of the quark fields, then the weak interactions violate CP invariance.

Mixing matrices appear in the broken sideways currents also. Typically, phases in these matrices cannot be absorbed by redefining fields, because both left-handed and right-handed fermions transform nontrivially under $G_S$. Sideways gauge bosons can couple to flavor-changing neutral currents. If the operator $C S d_1 d_2$ occurs in the effective interaction, Im $C$ must be suppressed. If $C$ is comparable to the coefficient of the term which generates the down quark mass, and $\arg C \approx 1$, this operator induces a CP-nonconserving part of $K^0 - \bar{K}^0$ mixing which is larger by $10^4$ than what is observed.

The effective Lagrangian below 1 TeV contains pseudo-Goldstone bosons whose couplings to fermions can be CP nonconserving. Exchange of these bosons can generate milliwatt CP nonconservation.

Spontaneous CP nonconservation typically does not occur if the sideways interaction is vectorial. But the sideways interaction must be nonvectorial to generate realistic quark masses. Toy models in which spontaneous CP nonconservation occurs will be exhibited in Ref. 15.

We have proposed a natural mechanism for generating CP nonconservation in a gauge theory without elementary scalar fields or fermion bare masses. Although the strong CP problem is not automatically solved, strong CP nonconservation may be avoided in particular models. However, CP nonconservation in the broken sideways interactions is unavoidable, if weak CP nonconservation occurs at all. Hence, if the electric dipole moment of the neutron is not discovered soon, the general approach to CP nonconservation which we advocate in this paper will become untenable.

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13. The sideways interactions are isolated from the CP-nonconserving interactions which generate the baryon asymmetry of the early universe. See M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978), and 42, 746B (1979).
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17. If $n_\rho = 2$ or 4, other forms occur, but our conclusions are not altered.
22. Rare processes such as $\mu^+ e^-$ and $\tau^- \mu^+$ are also induced by operators appearing in the effective Lagrangian at rates comparable to present experimental upper limits.