Ph 12c

Homework Assignment No. 5
Due: 5pm, Thursday, 12 May 2011

Do Problems 4, 6, 7, and 8 in Chapter 8 of Kittel and Kroemer, plus the two additional problems below.

For problem 8.7 in K&K, find the steady-state temperature inside the refrigerator assuming it operates reversibly and is perfectly insulated, and that the exterior temperature is 300°K.

1. Degenerate Fermi gas in two dimensions

(a) Consider an ideal non-relativistic Fermi gas at zero temperature in a two-dimensional square box with side length $L$. Express the Fermi wave number $k_F$ in terms of $L$ and the number of particles $N$. Assume that the fermions have spin $\frac{1}{2}$.

(b) Find the total internal energy $U$ of the gas at zero temperature, expressed in terms of $L$, $N$, and the particle mass $m$.

(c) Express the energy per particle $U/N$ in terms of the Fermi energy $\varepsilon_F$.

(d) Find the pressure $p$ of the gas at zero temperature, which in two dimensions is given by the derivative of $U$ with respect to the area $A = L^2$,

$$p = -\left(\frac{\partial U}{\partial A}\right)_{\sigma,N}.$$

Express $p$ in terms of $U$ and $A$.

2. Bose condensation in two dimensions

Consider an ideal gas of non-relativistic spin-0 bosons, at temperature $\tau$, in a two-dimensional box of side $L$.

(a) Find the two-dimensional density of states factor $D(\varepsilon)$.

(b) Express the activity $\lambda \equiv e^{\mu/\tau}$ in terms of $N_0$, the number of particles in the ground orbital. Use the convention that the energy of the ground orbital is $\epsilon_0 = 0$. 

c) Find $N_e(\tau)$, the number of particles in excited orbitals. You may assume that the box is big enough so that the sum over states can be replaced by an integral. Be sure to use the formula found in (b) for $\lambda$, not the $N_0 \to \infty$ limit of that formula. Your answer for $N_e$ will therefore be expressed in terms of $N_0$. **Hint:** $\int dx (ae^x - 1) = \ln(a - e^{-x})$.

d) Find the *two-dimensional* Einstein condensation temperature $\tau_E$. This is the smallest temperature such that, for $\tau > \tau_E$, the fraction $N_0/(N_0 + N_e)$ of particles in the ground orbital vanishes in the limit $L \to \infty$. (The limit is to be taken with the density $(N_0 + N_e)/L^2$ held fixed.)