Physics 236A, Fall 2002
Problem Set 8

Date assigned: November 22 **Due on December 6 at 2:30pm**

Announcement:
There will be no class in the week of November 25. This is the last homework problem set for this quarter. Since we have a two weeks deadline, there are slightly more problems in this set than usual.

Further Reading (Optional):

"Was Einstein Right?" Clifford M. Will (Basic Books, ISBN 0-465-09087-7).
This is a popular science book on experimental test of General Relativity.

For more *serious* reading, one may look at
"Theory and Experiment in Gravitational Physics," Clifford M. Will, (Cambridge U. Press, ISBN 0-521-31710-X).

Homework Problems:

[1] Show that, in 2 spacetime dimensions, the left hand side of the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu},$$

vanishes identically. This means that the gravitational dynamics in 2 spacetime dimensions is trivial and the Einstein equation simply impose the constraint $T_{\mu\nu} = 0$ on the matter field.

[2] The matter energy-momentum tensor $T_{\mu\nu}$ is covariantly conserved, $\nabla_\mu T^{\mu\nu} = 0$. By using this and the Einstein equation, show

$$\partial_\mu \left(\sqrt{-g} T^\mu_{\ \nu} \right) = \frac{1}{16\pi G} \sqrt{-g} \partial_\nu g_{\rho\sigma} (R^{\rho\sigma} - \frac{1}{2} g^{\rho\sigma} R).$$

For the weak gravitational field, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, show that the right-hand side is of order h^2.

[3] In the class, we started with a general spherically symmetric metric and showed that a solution to the vacuum Einstein equation, $R_{\mu\nu} = 0$, is always Schwarzschild

metric. Let us consider a simpler case when we assume that the metric is both static and spherically symmetric. So we have

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Let us try to compute $\Gamma_{\nu\rho}^{\ \mu}$ for this metric. Here is one efficient way to do it. The idea is that, if we know all geodesics obeying

$$\frac{d^2x^\mu}{dt^2} + \Gamma_{\nu\rho}^{\ \mu}\frac{dx^\nu}{dt}\frac{dx^\rho}{dt} = 0,$$

they uniquely determine all Γ coefficients.

(a) Show that, for the affine parametrization of a trajectory $x^\mu(t)$,

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^n = \text{const},$$

the extremal condition

$$\delta\int\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}dt = 0,$$

is equivalent to

$$\delta\int g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu dt = 0.$$

(b) Write down the third equation in (a) for the static sperically symmetric metric.

(c) Compare the result of (b) with the geodesic equation and find the Γ coefficients.

[4] A plane gravitational wave is characterized by two polarizations $C_{11} = -C_{22}$ and C_{12}, which are transverse to the direction of the gravitational wave momentum. In the class, we discussed effects of the gravitational wave on a set of point particles distributed along a circle in the $1-2$ plane when $C_{11} \neq 0$ and $C_{12} = 0$ and when $C_{11} = 0$ and $C_{12} \neq 0$.

(a) Consider a gravitational wave with $C_{12} = \sqrt{-1}C_{11}$. What is its effect on the point particles along the circle.

(b) Can you describe what happens in a more general situation when $C_{12} = \alpha C_{11}$ for some complex number α?

[5] As explained in the class, the energy carried away per unit time by gravitational radiation from a source with the quadrupole moment tensor q_{ij} is given by

$$P = \frac{1}{45} \sum_{i,j=1}^{3} \left(\frac{d^3 Q_{ij}}{dt^3}(t-R) \right)^2,$$

where

$$Q_{ij} = q_{ij} - \frac{1}{3}\delta_{ij} q.$$

Using this formula, solve the problems 8 and 9 of Chapter 4 of Wald.