Physics 236A, Fall 2002

Problem Set 4

Date assigned: October 25 **Due on November 1 by 2:30pm**

Reading Assignments:

Study Sections 2.2, 2.3 and 2.4 of Wald.

Homework Problems:

[1]

(a) By using the fact that tangent vector fields X and Y on a manifold M transform as contra-variant vectors, show explicitly that their commutator $[X,Y]^\mu$ also transforms as a contra-variant vector.

(b) For the vector field X, define the one-parameter family of transformation φ_t on M by

$$\frac{d}{dt}\varphi_t\left(f(p)\right)_{|t=0} = \left(Xf\right)(p), \qquad p \in M,$$

and $\varphi_t \circ \varphi_s = \varphi_{t+s}$. (We often write it as $\varphi_t = \exp(tX)$.) The Lie derivative $\mathcal{L}$ of the vector field Y with respect to the vector field X is defined by

$$\mathcal{L}_X Y(p) = \lim_{t\to 0}\left[(\varphi_{-t})_* X(\varphi_t(p)) - X(p)\right].$$

Show that it is given by the commutator,

$$\mathcal{L}_X Y(p) = [X,Y](p).$$

[2] Verify explicitly that, if ω_μ is a one-form (co-tangent vector), $\partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ transforms as a rank-2 covariant tensor.

[3] Do Problem 5 of Chapter 2 of Wald.

[4] Do Problems 8.a and 8.b of Chapter 2 of Wald.