Physics 236A, Fall 2002
Problem Set 1

Date assigned: October 4 **Due on October 11 by noon**

Bring the homework to 471 Lauritsen, attn: Lee/Park.

Reading Assignments:
Read Chapter One.

Homework Problems:

[1] Show

$$\epsilon_{lmn} R_{li} R_{mj} R_{nk} = \mathrm{det}R \ \epsilon_{ijk},$$

for any 3×3 matrix R.

[2] The Maxwell equations can be written as

$$\begin{aligned}\epsilon_{ijk}\partial_j E_k + \frac{1}{c}\partial_t H_i &= 0,\ \ \partial_i H_i = 0 \\ \epsilon_{ijk}\partial_j H_k - \frac{1}{c}\partial_t E_i &= \frac{4\pi}{c} J_i,\ \ \partial_i E_i = 4\pi\rho.\end{aligned}$$

Show that these equations are invariant under

$$\begin{aligned}x^i \to \tilde{x}^i &= R_{ij} x^j, \qquad R \in O(3) \\ E_i(x) \to \tilde{E}_i(\tilde{x}) &= R_{ij} E_j(x) \\ H_i(x) \to \tilde{H}_i(\tilde{x}) &= \mathrm{det}R \ R_{ij} H_j(x) \\ \rho(x) \to \tilde{\rho}(\tilde{x}) &= \rho(x),\ \ J_i \to \tilde{J}_i(\tilde{x}) = R_{ij} J_j(x).\end{aligned}$$

$\vec{E}(x)$ is a polar vector and $\vec{H}(x)$ is an axial vector.

[3] Prove the identity,

$$\epsilon_{ijk}\epsilon_{lmn} = \det \begin{pmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{pmatrix}.$$

Using this, show

$$\epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}.$$

[4] Solve the following equations for X_i and Y_i.

(a)

$$aX_i + \epsilon_{ijk} Y_j b_k = A_i, \quad cY_i + \epsilon_{ijk} X_j b_k = B_i.$$

(b)

$$aX_i + bY_i = P_i, \quad \epsilon_{ijk} X_j Y_k = Q_i \quad (P_i Q_i = 0).$$