Physics 129C, Spring 2002

Problem Set 7

Date assigned: May 16 **Due on May 23 at 1:00pm**

Announcement: There will be no class meeting on May 30.

Reading Assignment: Read Chapters 6 and 7 of Tung.

Bring the homework to 422 Downs, attn: Takuya Okuda and Xinkai Wu.

[1] Decompose a tensor product of the $j = 3/2$ and $j = 1$ representations of $SU(2)$ using the method described in the lecture. Compute the Clebsh-Gordan coefficients explicitly.

[2] The Campbell-Hausdorff formula enables you to express $e^{\epsilon X} e^{\epsilon Y}$ as in the form e^Z, where X, Y, and Z are matrices and ϵ is come constant. In the class, I gave a formula for Z to all order in the ϵ expansion. Compute $e^{\epsilon X} e^{\epsilon Y}$ and derive the formula formula for Z to the order ϵ^3.

[3]
(a) The orthogonal group $O(n)$ acts on the n-dimensional Euclidean space R^n. Show that the Lie algebra of $O(n)$ is generated by

$$L_{ab} = -i(e_{ab} - e_{ba}), \quad (a, b = 1, \cdots, n;\ a < b),$$

where e_{ab} is an $n \times n$ matrix such that its matrix element in the a-th column and the b-th row is 1 and all the other matrix elements are zero.

(b) Compute the structure constant of $O(n)$ in the basis of L_{ab} defined in the above.