Physics 129C, Spring 2002

Problem Set 6

Date assigned: May 9 **Due on May 16 at 1:00pm**

Announcement: There will be no class meeting on May 30.

Reading Assignment: Read Chapters 6 and 7 of Tung.

Bring the homework to 422 Downs, attn: Takuya Okuda and Xinkai Wu.

[1] Consider the Young tableau with 3 boxes corresponding to the partition 3 = 2+1. There are 6 ways to put integers from 1 to 3 in the boxes. Find states in the regular representation of S_3 associated to this Young tableau, and show that there are only 2 linearly independent states. Verify that these two states make the representation E of $C_{3v} \simeq S_3$.

[2] Display all the Young tableaux of the group S_4. Enumerate all the inequivalent irreducible representations of S_4 and specify their dimensions.

[3] The Cartan metric g_{AB} of a Lie algebra is defined by $g_{AB} = C^C_{AD} C^D_{BC}$, where C^C_{AB} is the structure constant with respect to generators L_A $(A = 1, \cdots, d)$ of the Lie algebra.

(3-1) Show that $C_{ABC} = C^D_{AB} g_{DC}$ is totally antisymmetric in the indices A, B, C.

(3-2) Suppose g_{AB} is invertible and denote the inverse by g^{AB}, *i.e.*, $g^{AB} g_{BC} = \delta^A{}_C$. Show that $\hat{C}^{(2)}$ defined by $C^{(2)} = g^{AB} L_A L_B$ commutes with all the elements L_A of the Lie algebra.