Physics 129C, Spring 2002

Problem Set 4

Date assigned: April 25 **Due on May 2 at 1:00pm**

Announcement: There will be a make-up class on April 29, 4:30 - 5:30.

Reading Assignment: Chapter 3 and Chapter 5 of Tung.

Bring the homework to 422 Downs, attn: Takuya Okuda and Xinkai Wu.

[1] Consider the symmetry group of the equilateral triangle. When the coordinates undergo some transformation of the group, the quadratic form

$$a\frac{x^2}{\sqrt{2}} + bxy + c\frac{y^2}{\sqrt{2}}$$

is transformed into another quadratic form with different a, b, c which are related to the original ones by a 3×3 matrix. The 3×3 matrices so obtained form a representation of the group. Find the characters of this representation.

[2] Do the following matrices form a group?

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where $\omega = e^{2\pi i/3}$ and the group multiplication is defined as the matrix multiplication. If they don't, find other 2×2 matrices needed to complete a group with smallest order possible. Divide the elements of the group into conjugacy classes.

[3] Consider a representation of a group G which may not be faithful. Show that the set H of elements of G which map on to the unit matrix in the representation is an invariant subgroup of G. Show that this representation gives a faithful representation for the coset group G/H.