1. Consider a surface $S$ in $\mathbb{R}^3$ given by the equation $z = f(x, y)$. The standard flat metric on $\mathbb{R}^3$ induces a curved metric on this surface. It also gives rise to a second fundamental form.

(a) Express the metric of $S$ at a point $(x, y)$ in terms of $f(x, y)$ and its derivatives.

(b) Express the second fundamental form of $S$ in terms of $f(x, y)$.

2. Show that the isomorphism of the Lie algebras of $SO(n)$ and $Spin(n)$ maps an antisymmetric matrix $a_{ij}$ (regarded as an element of the Lie algebra of $SO(n)$) to

$$
\frac{1}{4} \sum_{i,j} a_{ij} e_i \circ e_j,
$$

where $e_i$ is an element of an orthonormal basis of $\mathbb{R}^n$, regarded as an element of $Cl(n)$. 

Week 2 (due April 15)