Homework 5

Duality: Consider a Wilson line operator $W_q(\gamma)$ in four-dimensional $U(1)$ gauge theory with $\gamma = \partial D$,

$$W_q(\gamma) = \exp\left(iq \oint_\gamma A\right) = \exp\left(iq \int_D F\right)$$

Show that $W_q(\gamma)$ is dual to 't Hooft line operator (disorder operator) $T_q(\gamma)$ defined by the following prescription: remove γ from the space-time manifold M and consider a path integral over gauge field configurations on $M \setminus \gamma$, such that

$$\int_{\mathbf{S}^2} \tilde{F} = 2\pi q$$

where $\mathbf{S}^2$ is a 2-sphere linked with γ in M (a non-trivial 2-cycle in $M \setminus \gamma$).