New Paths in the String Theory Landscape

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Based on
U. Danielsson, N. Johansson and M.L., hep-th/0612222

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**Motivation**

String theory lives in 10D, we live in 4D.

- Compactify.
- Fluxes.
- Branes.

⇒ string theory landscape of vacua
Motivation

Natural questions:
How many vacua?
Distribution?
Continuously connected?
Barriers?

Effects from topography
Tunneling, domain walls, inflation, finiteness...
Flux compactifications 1

Flux vacua

Ingredients: manifolds, fluxes, branes... enormous landscape!

A landscape model

Type IIB SUGRA on (conformal) CY 3–fold.

Calabi–Yau manifolds

Complex, Kähler, Ricci flat.
$h^{2,1}$ Complex structure (CS) moduli.
$h^{1,1}$ Kähler moduli.
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Type IIB SUGRA on (conformal) CY 3–fold.

Calabi–Yau manifolds
Complex, Kähler, Ricci flat.
$h^{2,1}$ Complex structure (CS) moduli.
$h^{1,1}$ Kähler moduli.
Fixing the complex structure

Complex structure $\sim$ holomorphic 3-form $\Omega(z)$.

3–cycles

3-cycles basis $A_I, B_J$.

$$\int_{A_i} \alpha_J = \int_{B_i} \beta_J = \int \alpha_I \wedge \beta_J = \delta_{IJ}$$

- Periods $\Pi_i(z) = \int_{C_i} \Omega(z)$
- $z$: CS moduli
- $\Pi(z) = (\Pi_1(z), \Pi_2(z), \ldots \Pi_N(z))$

3–flux

- IIB: RR $F$ and NS $H \Rightarrow G = F - \tau H$
- Quantized:
  $$\int_{C_i} F \sim F_i, \int_{C_i} H \sim H_i, \quad F_i, H_i \in \mathbb{Z}$$
- D3 tadpole condition:
  $$\int_{CY} F \wedge H = N_{D3}$$
The potential for CS moduli

- **Fluxes** wrapping non-trivial cycles $\rightarrow$ potential $V$.

- $V = e^K (\|DW\|^2 - 3|W|^2)$
  - Kähler potential $e^K = \frac{1}{\text{Im}(\rho)} \Pi^\dagger \cdot Q \cdot \Pi$
  - Superpotential $W = G \cdot \Pi(z)$

- CS moduli and $\tau$ fixed at minima of potential.
- No-scale: Kähler moduli unfixed perturbatively.
Paths between vacua

Paths between flux vacua (hep-th/0612222)

CS moduli space is complicated:
- singularities, branch cuts, non-trivial loops
- monodromies of 3-cycles.

Idea: Use monodromies to continuously connect vacua.
Paths between vacua

Monodromies

- Period monodromies \( \Pi(z) \rightarrow T \cdot \Pi(z) \)
- \( T \in \mathcal{M} \subset Sp(N, \mathbb{Z}) \)

E.g. **Mirror Quintic**
- \( h^{2,1} = 1 \) CS modulus
- \( h^{1,1} = 101 \) Kähler moduli
Recall: \( V = e^K \left( ||DW||^2 - 3|W|^2 \right), \ W = G \cdot \Pi(z) \)

Thus \( \Pi(z) \rightarrow T \cdot \Pi(z) \)

\( \rightarrow V \) has branch cuts in CS moduli space.

Traverse cuts \( \rightarrow \) paths between minima.

\( \Pi \rightarrow T \cdot \Pi \) or \( G \rightarrow G \cdot T \)

\( T \in \mathcal{M} \subset Sp(N, \mathbb{Z}) \rightarrow \)

\( \int F \wedge H \) unchanged.
Several **continuously connected minima** found:

No **infinite** series of minima found.

What about flux minima **not** related by monodromies ("islands")?
Extend moduli space

An extended landscape model 0710.0620

Monodromies: important for topography.
Larger moduli space ⇒ more monodromies.

Geometric transitions:
Moduli spaces of different Calabi–Yau 3-folds are connected

Idea: extend $\mathcal{M}_{101,1}$ CS moduli space. Connect it to what?

\[
\begin{align*}
\mathcal{M}_{1,101} & \quad \text{Mirror} \\
\mathcal{M}_{101,1} & \quad \text{Mirror} \\
\mathcal{M}_{2,86} & \quad \text{Mirror} \\
\mathcal{M}_{86,2} &
\end{align*}
\]
Geometric transitions

Mirror symmetry $\rightarrow \mathcal{M}_{(86,2)} \xleftarrow{GT} \mathcal{M}_{(101,1)}$

- $\mathcal{M}_{(86,2)}$ shrinks 16 3-cycles $A_i$
- $A_1 - A_2 = \delta D_1$
- ...$
- A_{15} - A_{16} = \delta D_{15}$

- $\mathcal{M}_{(101,1)}$ blow up 16 2-cycles $a_i$
- $\sum a_i = \delta B$
- $\delta D_i = 0$
Motivation
Flux compactifications
Paths between vacua
Extend moduli space
Geometric transitions with fluxes
Infinite series of minima
Conclusions and Outlook

Extend moduli space
Extend moduli space

We need to:

- Construct $M_{(86,2)}$ (using toric geometry)

- Compute **periods** of $M_{(86,2)}$

- **Embed** $M_{(101,1)}$ in $M_{(86,2)}$

- Compute new **monodromies**

Geometric transitions with flux?
Infinite series of string theory vacua?
Toric geometry

Construct CY: zero locus of polynomial equations on toric variety.
Toric variety: \( \frac{(\mathbb{C}^*)^n - \mathbb{Z}}{G} \)
Fans and polytopes \( \leftrightarrow \) toric variety and equations.

Batyrev’s mirror construction

\( M_{(2,86)} \): Cl in \( \mathbb{P}^1 \times \mathbb{P}^4 \) \( \Rightarrow \) Polytope for \( M_{(2,86)} \): \( \nabla = \nabla_1 + \nabla_2 \)
Mirror construction:
Polytope for \( M_{(86,2)} \): \( \Delta = \Delta_1 + \Delta_2, \langle \nabla_k, \Delta_j \rangle \geq -\delta_{k,j} \)

\( M_{(86,2)} \)

\[ f_1 \equiv 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5 / t_2 t_3 t_4 t_5 \]
\[ f_2 \equiv 1 - a_6 / t_1 - a_7 t_2, \]
CS moduli \( \sim a_i \):
\[ \phi_1 = a_1 a_6, \phi_2 = a_2 a_3 a_4 a_5 a_7. \]
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\( \mathcal{M}_{(86,2)} \)

\( f_1 = 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5 / t_2 t_3 t_4 t_5 \)
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\(\mathcal{M}_{(2,86)}\): CI in \(\mathbb{P}^1 \times \mathbb{P}^4\) ⇒ Polytope for \(\mathcal{M}_{(2,86)}\): \(\nabla = \nabla_1 + \nabla_2\)
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\(\mathcal{M}_{(86,2)}\)

\(f_1 \equiv 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5 / t_2 t_3 t_4 t_5\)
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CS moduli \(\sim a_i: \phi_1 = a_1 a_6, \phi_2 = a_2 a_3 a_4 a_5 a_7.\)
The fundamental period

$$\omega_0 = \frac{1}{(2\pi i)^5} \int_\gamma \frac{1}{f_1 f_2} \frac{dt_1}{t_1} \wedge ... \wedge \frac{dt_5}{t_5}$$

Near $\phi_1 = \phi_2 = 0$

$$\omega_0(\phi) = \sum_{n_1, n_2} c(n_1, n_2) \phi_1^{n_1} \phi_2^{n_2}, \text{ where } c(n_1, n_2) = \frac{(n_1+4n_2)! (n_1+n_2)!}{(n_1)!^2 (n_2)!^5}$$

Picard–Fuchs equations

Recall $\Pi_i = \oint_{C_i} \Omega$

$H^3 = H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$ is finite

$\Rightarrow L_k \Omega = d\eta$

$\Rightarrow L_k \Pi_i = \oint_{C_i} L_k \Omega = \oint_{C_i} d\eta = 0$

We get: 2 DE of degree 2 and 3

$\rightarrow 6$ linearly indep. solutions $\sim 6$ periods.
The fundamental period

$$\omega_0 = \frac{1}{(2\pi i)^5} \int_\gamma \frac{1}{f_1 f_2} \frac{dt_1}{t_1} \wedge \ldots \wedge \frac{dt_5}{t_5}$$

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We get: 2 DE of degree 2 and 3

$\rightarrow$ 6 linearly indep. solutions $\sim$ 6 periods.
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\[ M_{(86,2)} \] periods: 2

**The Frobenius method**

\( \phi_1 = \phi_2 = 0 \) regular singular point

\( \rightarrow \) five solutions with logarithmic singularities:

Using \( \omega(\rho, \phi) = \sum_{n_1, n_2} c(n_1 + \rho_1, n_2 + \rho_2) \phi_1^{n_1+\rho_1} \phi_2^{n_2+\rho_2} \)

we get all periods as: (hep-th/9406055)

\[
\begin{align*}
\omega_1 &= \partial_{\rho_1} \omega \big|_{\rho=0}, \\
\omega_2 &= \partial_{\rho_2} \omega \big|_{\rho=0}, \\
\omega_3 &= \kappa_{1jk} \partial_{\rho_j} \partial_{\rho_k} \omega \big|_{\rho=0}, \\
\omega_4 &= \kappa_{2jk} \partial_{\rho_j} \partial_{\rho_k} \omega \big|_{\rho=0}, \\
\omega_5 &= \kappa_{ijk} \partial_{\rho_i} \partial_{\rho_j} \partial_{\rho_k} \omega \big|_{\rho=0}
\end{align*}
\]

\( \kappa_{ijk} = \int J_i \wedge J_j \wedge J_k \) classical intersection numbers.

Want integral and symplectic monodromies: change basis.
(No details here).
**Embed $M_{(101,1)}$ in $M_{(86,2)}$**

**$M_{(86,2)}$**

Recall: 

$$f_1 ≡ 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5/t_2 t_3 t_4 t_5 = 0$$

$$f_2 ≡ 1 - a_6/t_1 - a_7 t_2 = 0.$$ 

**$M_{(101,1)}$**

Substitute $f_2$ into $f_1$

$$b_0 + b_1 u_1 + b_2 u_2 + b_3 u_3 + b_4 u_4 + \frac{b_5}{u_1 u_2 u_3 u_4} + \frac{b_6}{u_1 u_2 u_3} = 0$$

Redefined CS moduli

$$z_1 = \frac{b_1 b_2 b_3 b_6}{b_0^4} = \frac{\phi_2}{(1-\phi_1)^4} \quad \text{and} \quad z_2 = -\frac{b_1 b_2 b_3 b_4 b_5}{b_0^5} = \frac{\phi_1 \phi_2}{(1-\phi_1)^5}$$

Take $z_1 \rightarrow 0$: Mirror quintic equation!
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Embed $\mathcal{M}_{(101,1)}$ in $\mathcal{M}_{(86,2)}$

**$\mathcal{M}_{(86,2)}$**

Recall: $f_1 \equiv 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5 / t_2 t_3 t_4 t_5 = 0$

$f_2 \equiv 1 - a_6 / t_1 - a_7 t_2 = 0$.

**$\mathcal{M}_{(101,1)}$**

Substitute $f_2$ into $f_1$

$b_0 + b_1 u_1 + b_2 u_2 + b_3 u_3 + b_4 u_4 + \frac{b_5}{u_1 u_2 u_3 u_4} + \frac{b_6}{u_1 u_2 u_3} = 0$

Redefined CS moduli

$z_1 = \frac{b_1 b_2 b_3 b_6}{b_0^4} = \frac{\phi_2}{(1-\phi_1)^4}$ and $z_2 = -\frac{b_1 b_2 b_3 b_4 b_5}{b_0^5} = \frac{\phi_1 \phi_2}{(1-\phi_1)^5}$

Take $z_1 \to 0$: Mirror quintic equation!
The mirror quintic locus

The MQ locus $z_1 \to 0$: Which period vanish?
Monodromy around the locus?

Analytically continue $\omega_0 \Rightarrow$

$$\omega_0 = \sum_{m_1,m_2=0}^{\infty} \frac{(4m_1+5m_2)!}{((m_1+m_2)!)^3 m_1!(m_2!)^2} Z_1^{m_1} Z_2^{m_2}.$$  

$z_1 \to 0$: MQ fundamental period.
Other periods:
Analytically continue $\omega_i$: focus on derivatives.
The mirror quintic locus

The MQ locus $z_1 \to 0$:
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$z_1 \to 0$: MQ fundamental period.
Other periods:
Analytically continue $\omega_i$: focus on derivatives.
Embedded periods and monodromies

**Periods**

Integral and symplectic basis:

\[ \Pi_{(86,2)} = \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \\ \Pi_5 \\ \Pi_6 \end{pmatrix} \rightarrow \begin{pmatrix} \Pi_1^{MQ} \\ \Pi_2^{MQ} \\ \Pi_3^{MQ} \\ \Pi_4^{MQ} \\ 0 \\ \sum c_i \Pi_i^{MQ} \end{pmatrix} \]

**New paths between vacua**

4 new monodromies.
Geometric transitions.
→ New series of MQ vacua
**Geometric transitions with fluxes**


Need to be careful:

\[ M_{(86,2)} \] monodromy might yield flux through *A* or *B*!

**Flux through A**  hep-th/9811131, 0008142...

RR/NS–flux through **shrinking** 3–cycle *A*:

→ D5/NS5–branes on new 2–cycles.

Positions of 5–branes ∼ new **open string moduli**.

← New period: \[ \Pi_B(t, z) = \int_B \Omega \rightarrow V_{MQ}(z) \rightarrow \tilde{V}_{MQ}(t, z) \].
Geometric transitions with fluxes

Flux through $B$ 0709.4277, hep-th/0510042

RR/NS Flux through torn 3-cycle $B \rightarrow$ D1/F1-instantons?
No new terms in the MQ potential.
### Geometric transitions with fluxes

#### Flux through both 3–cycles $A$ and $B$

- **New open string moduli**
- Tadpole condition:
  \[ \int F \wedge H \text{ might change } \rightarrow \text{ D3–branes.} \]

#### Examples


Flux potential at transition

Near transition point: $V_{(86,2)}(z_1, z_2) = V_1(z_1, z_2) + V_2(z_2)$;
$V_2(z_2) \xrightarrow{z_1 \to 0} V_{MQ}(z)$
With flux through $A$: $V_1(z_1, z_2) \xrightarrow{z_1 \to 0} \infty$
Without flux through $A$: $V_1(z_1, z_2) \xrightarrow{z_1 \to 0} 0$

No flux through shrinking cycle $\rightarrow$ geometric transition controlled.
Look for connected minima without such flux.
Infinite series of minima

Requirements

Apply monodromy \( n \) times:
\[
F_0 \rightarrow F_0 T^n
\]
If \( T = 1 + \Theta \), \( \Theta^2 = 0 \)
\[
\leftrightarrow F_0 T^n = F_0 + nF_0 T
\]
Start flux \( F_0, H_0 \)

- \( F_0 T = F_L \)

Limit flux \( F_L, H_L \)

- has minimum
- \( F_L \wedge H_L = 0 \)
- \( F_L T = F_L \)

\[
F_0 = F_0 + nF_L, \ H_0 = H_0 + nH_L \Rightarrow \text{infinite number of minima.}
\]
N.B. Kähler moduli not fixed.
Conclusions and Outlook

Semi-discrete landscape.
- Topography → dynamics.
- Monodromies connect vacua.
- New, continuous paths.

The new paths allow us to
- connect more vacua continuously.
- find infinite series of minima.
- describe domain walls.
- use connected moduli spaces.

Outlook
- Kähler moduli dynamics. Back reaction.
- Transition with fluxes.
- Tunnelling between minima.
- Inflation.