Global Warming

Sunlight passes through atmosphere, intensity \( I_s \) absorbed by earth (reflected light not included in \( I_s \)).

Earth radiates in IR with intensity \( I_e \); a fraction \( a \) of this IR radiation is absorbed by atmosphere.

Atmosphere radiates intensity \( I_a \) both into space and down to earth. Earth absorbs radiation from atm.

Earth in equilibrium: \( I_e = I_s + I_a \)

(absorbed intensity matches radiated intensity).

Intensity emerging from atm matches intensity incident from sun:

\[ I_s = I_a + (1-a)I_e \]

Eliminate \( I_a \):

\[ I_s = (I_e - I_s) + (1-a)I_e = (2-a)I_e - I_s \]

\[ \Rightarrow I_e = \left( \frac{2}{2-a} \right)I_s \]

Limiting cases: \( a = 0 \Rightarrow I_e = I_s \) (earth radiates at rate it absorbs.)

\( a = 1 \Rightarrow I_e = 2I_s \) (Now radiation from atm matches solar intensity, on earth, equal intensity from sun and atm.)

\( a \) is increasing as \( \text{CO}_2 \) in atm increases \( \Rightarrow \) earth warms. We know

\( a \approx 0.75, \ a_{\text{CO}_2} \approx 0.07, \ a_{\text{CO}_2} \propto \text{concentration of CO}_2 \)
\[ N = 400 \text{ ppm} \]
\[ \Delta N = 2 \text{ ppm/yr} \implies \Delta T_E \approx 0.02^\circ/\text{yr} \]

**Why?**

\[ I_E = 6B \frac{1}{T_E^4} \implies T_E \propto \left( \frac{2}{2-a} \right)^{\frac{1}{4}} \]

\[ \implies \Delta T_E = \Delta a \frac{1}{4} (2-a)^{\frac{5}{4}} \frac{1}{T_E} \approx \frac{1}{5} \Delta a \frac{T_E}{2} \quad (a \approx 0.75) \]

\[ \Delta a_{\text{CO}_2} = a_{\text{CO}_2} \left( \frac{\Delta a}{a} \right)_{\text{CO}_2} \]

\[ = 0.07 \frac{2 \text{ ppm}}{400 \text{ ppm}} \]

\[ \implies \Delta T_E = \frac{1}{5} (0.07) \frac{2}{400} \left( T_E \approx 300^\circ \text{K} \right) \]

\[ \approx (7 \times 10^{-5}) (300) \approx 0.02^\circ/\text{yr} \quad \left( ^\circ \text{C} \right) \]

*Earth warms 1°C in 50 years, assuming \( \Delta N = 2 \text{ ppm/year} \) and \( a_{\text{CO}_2} \propto N_{\text{CO}_2} \).

*(Of course, model is vastly oversimplified...)*