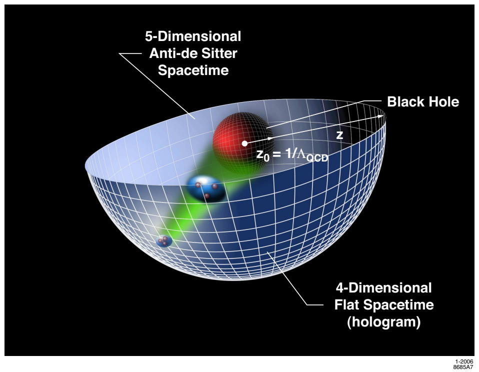
Holographic Construction of States, Form Factors, and the Hadron Spectrum in AdS/QCD

Guy F. de Téramond

Universidad de Costa Rica and SLAC

In Collaboration with Stan Brodsky



Caltech High Energy Physics Seminar

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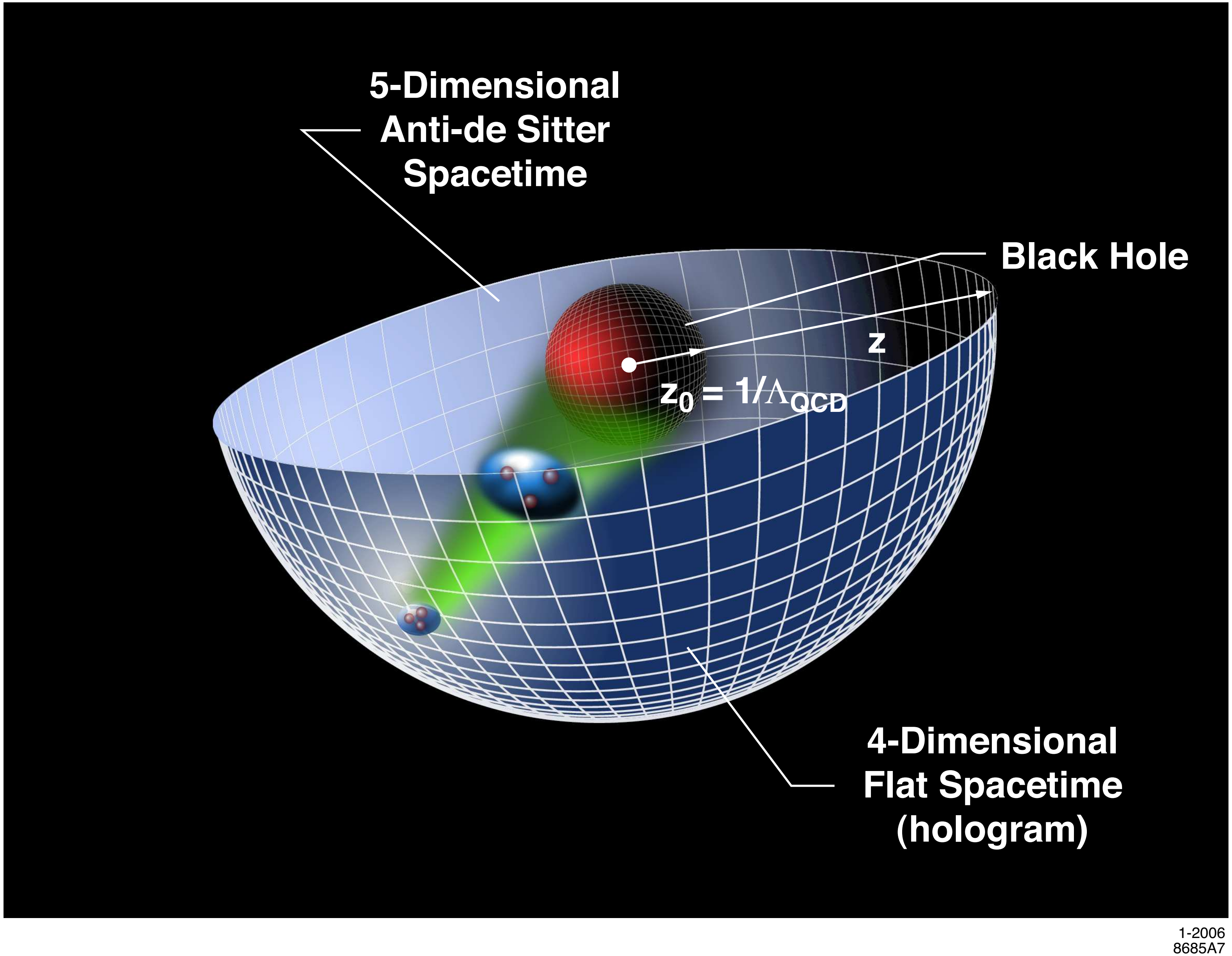
Motivation: AdS/QCD Correspondence

- Strings describe extended objects (no quarks), QCD degrees of freedom are pointlike particles: how can they be related?
- More precisely: how can we map string states into partons?
- Precise mapping for strongly coupled QCD in the conformal limit
- Infinite tension limit of strings $\longrightarrow$ effective gravity description
- Holographic duality requires a higher dimensional warped space. Space with negative curvature and a 4-dim boundary: AdS_5
- Eigenvalues of normalizable modes inside AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for distribution of quarks at a given scale
- Non-normalizable modes are related to external currents: they probe the cavity interior


5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)
1-2006
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Outline

- The Holographic Correspondence
- ADS/CFT and QCD
- Strongly Coupled Conformal QCD and Holography: Scale Transformations and Confinement
- Classical Correspondence and Interpolating Operators
- Quantum Fluctuations and Boundary Excitations
- Meson Spectrum
- Baryon Spectrum
- Hadronic Form Factors in AdS/QCD: Mesons and Baryons
- Holographic Model for Light-Front Wavefunctions
- Exact Holographic Mapping for Light-Front n-Parton State
- Outlook

The Holographic Correspondence

- Original correspondence between $\mathcal{N}=4$ SYM at large N_C and the low energy supergravity approximation to Type IIB string on $AdS_5 \times S^5$: Maldacena, hep-th/9711200.

$$
\begin{array}{ccc}
\text{Warped higher dim space} & & \text{Conformal d} = 4 \text{ spacetime boundary} \\
\text{Type IIB } (\text{AdS}_5 \times \text{S}^5) & \leftrightarrow & \mathcal{N} = 4 \text{ SYM } (\text{SO}(4,2) \otimes \text{SU}(4)) \\
? & \leftrightarrow & \text{QCD}
\end{array}
$$

$SO(4,2)$ is isomorphic with the isometries of AdS^5, and $SU(4) \sim SO(6)$ with S^5.

- <u>Description of strongly coupled gauge theory using a dual gravity description!</u>

- QCD is fundamentally different from SYM theories where all matter is in the adjoint rep of $SU(N_C)$. Introduction of quarks in the fundamental rep is dual to an open string sector: Gross and Ooguri, hep-th/9805129; E. Witten, hep-th/9805112.

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^μ, D, K^μ, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops. For $\beta = d\alpha_s(Q^2)/dQ^2$, QCD is a conformal theory: Parisi, Phys. Lett. B **39**, 643 (1972).

- Growing theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point: von Smekal, Alkofer and Hauck, arXiv:hep-ph/9705242; Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Deur, Burkert, Chen and Korsch, hep-ph/0509113 . . .

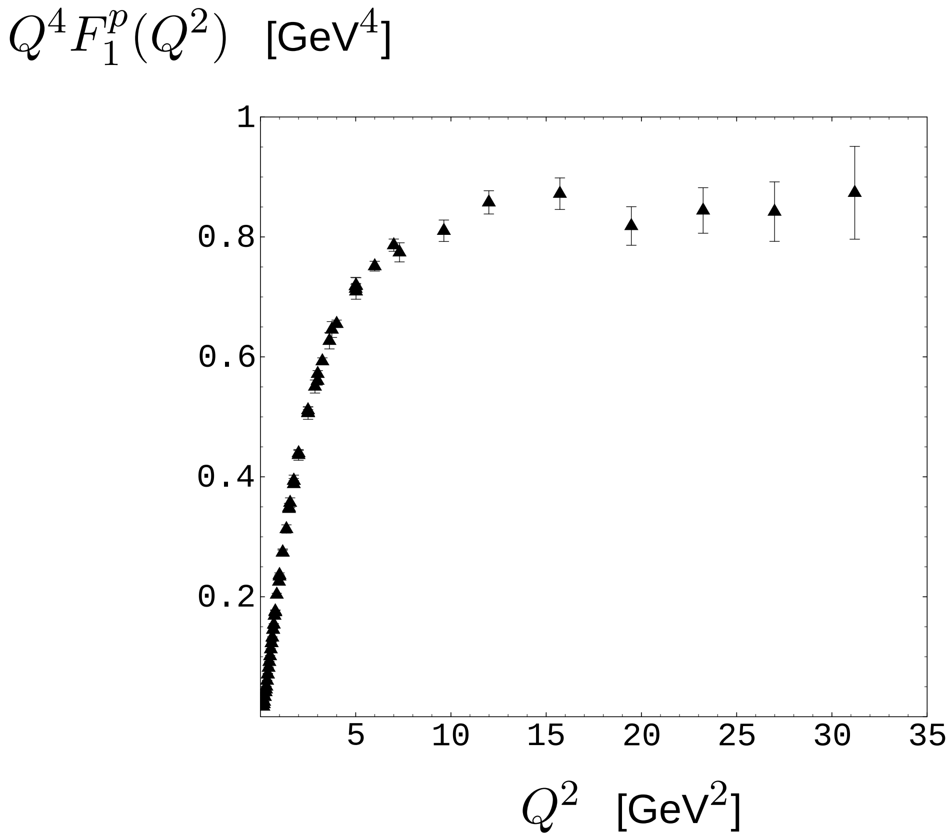
- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies Brodsky and Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Matveev *et al.*, Lett. Nuovo Cim. **7**, 719 (1973).

Conformal Behavior: Examples

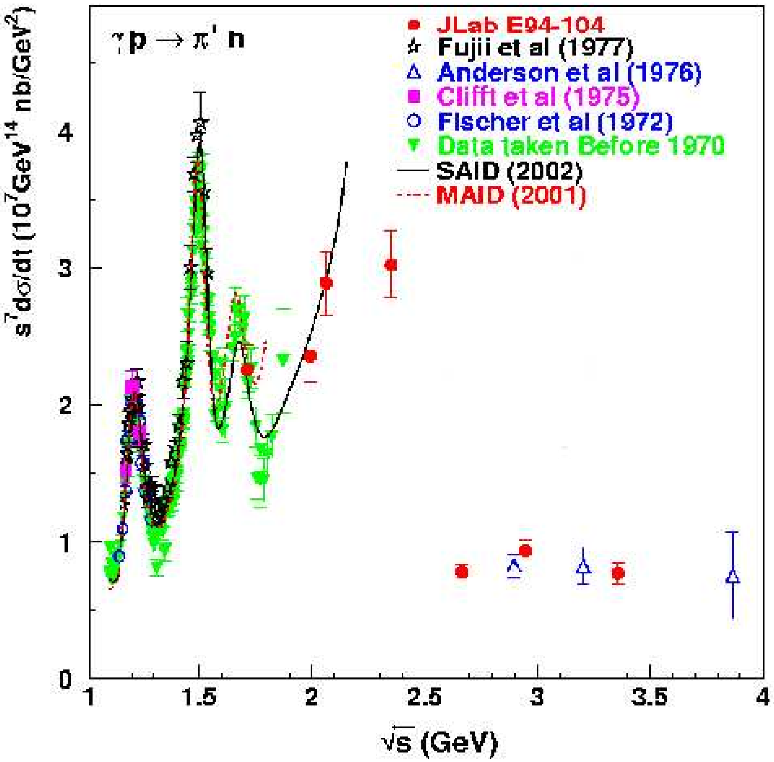
- Dirac proton form factor: $F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \;\; n = 3$



From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Pion photoproduction $\gamma + p \to \pi^+ + n$:

$$d\sigma/dt \sim 1/s^{n-2}, \;\; n = 1 + 3 + 2 + 3 = 9$$



From: Zhu *et al.* [Jefferson Lab Hall A Collaboration], nucl-ex/0409018.

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space $SO(1,5)$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Confinement

- QCD is a confining theory in the infrared with mass gap Λ_{QCD} and a well-defined spectrum of color-singlet states.
- There is a maximum separation of quarks and a maximum value of z.
- AdS space should end at a finite value $z_0 = 1/\Lambda_{\rm QCD}$.
- Cutoff at z_0 breaks conformal invariance and allows the introduction of the QCD scale.
- Non-conformal metric dual to a confining gauge theory (Polchinski and Strassler):

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) + ds^2_X,$$

  where $A(z) \to 0$ as $z \to 0$. Asymptotically: $AdS \times X$.

  (X is a 5-dim compact manifold if 4-dim gauge theory is dual to a critical (10-dim) string)

- Simplified model: metric factor $e^{2A(z)}$ is a step function. Analog to the MIT bag model, but with boundary conditions on the holographic coordinate.
- Truncated AdS/CFT model: conformal behavior at short distances and confinement at large distances.

Classical Correspondence and Interpolating Operators

- Duality between string in AdS_{d+1} and $N_C \to \infty$ limit of a conformal gauge theory given by the generating functionals of the string and CFT theories at the AdS d-dim boundary.

- d-dim generating functional in presence of external source Φ_o

$$Z_{QCD}[\Phi_o(x)] = \int \mathcal{D}[\psi, \overline{\psi}, A] \exp \left\{ iS_{QCD} + i \int d^d x \Phi_o \mathcal{O} \right\},$$

with $\mathcal{O}$ the hadronic interpolating operator.

- $d+1$-dim gravity partition function:

$$Z_{grav}[\Phi(x,z)] = \int \mathcal{D}[\Phi] e^{iS_{grav}[\Phi]} \, .$$

- Boundary condition:

$$Z_{grav}\left[\Phi(x, z=0) = \Phi_0(x)\right] = Z_{QCD}\left[\Phi_0\right].$$

Gubser, Klebanov and Polyakov, hep-th/9802109; Witten, hep-th/9802150.

- Near the boundary of AdS_{d+1} space $z \to 0$:

$$\Phi(x,z) \to z^{\Delta}\Phi_+(x) + z^{d-\Delta}\Phi_-(x).$$

- $\Phi_-(x)$ is the boundary source: $\Phi_- = \Phi_0$ (Non-normalizable mode).
- $\Phi_+(x)$ is the response function (normalizable mode):

$$\langle \mathcal{O} \rangle_{\Phi_0} = (2\Delta - d)\, \Phi_+(x),$$

Klebanov and Witten, hep-th/9905104.

- The physical AdS modes $\Phi(x,z) \sim e^{-iP\cdot x} f(z)$, are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For large-z $f(z) \sim z^{\Delta}$. The dimension Δ of the string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

- Interpolating operators $\mathcal{O}(x)$ at the boundary: $\mathcal{O} = G^a_{\mu\nu} G^{a\mu\nu}, \mathcal{O} = \overline{\psi}\gamma_5\psi, \ldots$
- QCD/gravity duality dictionary:

QCD (4-dim)	Gravity (5-dim)
Hadron int. op. $\mathcal{O}$	Normalizable mode $\Phi(x, z)$
Hadron mass $\mathcal{M}$	Eigenvalues of 5-dim WF
Conformal dim Δ	5-dim mass μ
Large Coupling	Small coupling
Large Q	Small z
Mass gap Λ_{QCD}	Cutoff $z = z_o$

Quantum Fluctuations and Boundary Excitations

- Higher Fock components of a hadron wf and states with non-zero orbital angular momentum are manifestations of quantum fluctuations of QCD.

- Metric fluctuations of the bulk geometry about the the fixed AdS background should correspond to quantum fluctuations of Fock states above the valence state.

- Higher orbital excitations are matched quanta to quanta with fluctuations around the spin $0, \frac{1}{2}, 1, \frac{3}{2}$ string solutions on AdS_5.

- The large-z asymptotic behavior of each mode is matched with the conformal dimension of the boundary interpolating operators for each hadron state, maintaining conformal invariance: an L quantum excitation corresponds to a five dimensional mass μ in the bulk.

- Allowed values of μ determined asymptotically requiring that the dimensions are spaced by integers: spectral relation $(\mu R)^2 = \Delta(\Delta - d) = L(L + d)$.
  p-form: $(\mu R)^2 = (\Delta - p)(\Delta - d + p) = L(L + d - 2p)$

Meson Spectrum

- Vector meson interpolating operator with twist-dimension minus spin-two, and conformal dimension $\Delta = 3 + L$

$$\mathcal{O}^{\mu}_{3+L} = \overline{\psi}\gamma^{\mu} D_{\{\ell_1} \dots D_{\ell_m\}}\psi.$$

- AdS wave equation with effective 5-dim mass μ. Solution is a vector field Φ_μ with polarization along Poincaré coordinates:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0,$$

with $\Phi_\mu(x,z) = e^{-iP\cdot x}\, f_\mu(z)$ and $P_\mu P^\mu = \mathcal{M}^2$ ($\Phi_z = 0$ gauge).

- Normalizable AdS vector mode:

$$\Phi_\mu(x,z) = C e^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}\left(z\mathcal{M}\right)\epsilon_\mu.$$

with $\Delta = d - 1 + L$ and $(\mu R)^2 = L(L + d - 2)$.

Introduction of Twist (Spin 0 and 1 AdS Modes)

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n} \sigma_i$.
- For a two quark state $\Delta \to \Delta - 1$. Change compensated in μ by the shift $L \to L - 1$.
- Lowest state corresponds to $(\mu R)^2 = -1$. Thus $-1 \leq (\mu R)^2$: Breitenlohner-Freedman stability bound for a 1-form.
- Two-quark vector meson described by wave equation (d=4)

$$\boxed{\left[z^2\,\partial_z^2 - 3z\,\partial_z + z^2\mathcal{M}^2 - L^2 + 4\right] f_\mu(z) = 0}$$

with solution

$$\Phi_\mu(x,z) = Ce^{-iP\cdot x} z^2 J_L(z\mathcal{M})\,\epsilon_\mu.$$

- Same equation for $\Delta = 4$, $\tau = 2$ glueball 0-form with $-4 \leq (\mu R)^2$ and solution

$$\Phi(x,z) = Ce^{-iP\cdot x} z^2 J_L(z\mathcal{M}).$$

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$.

- Normalizable AdS modes $\Phi(z)$

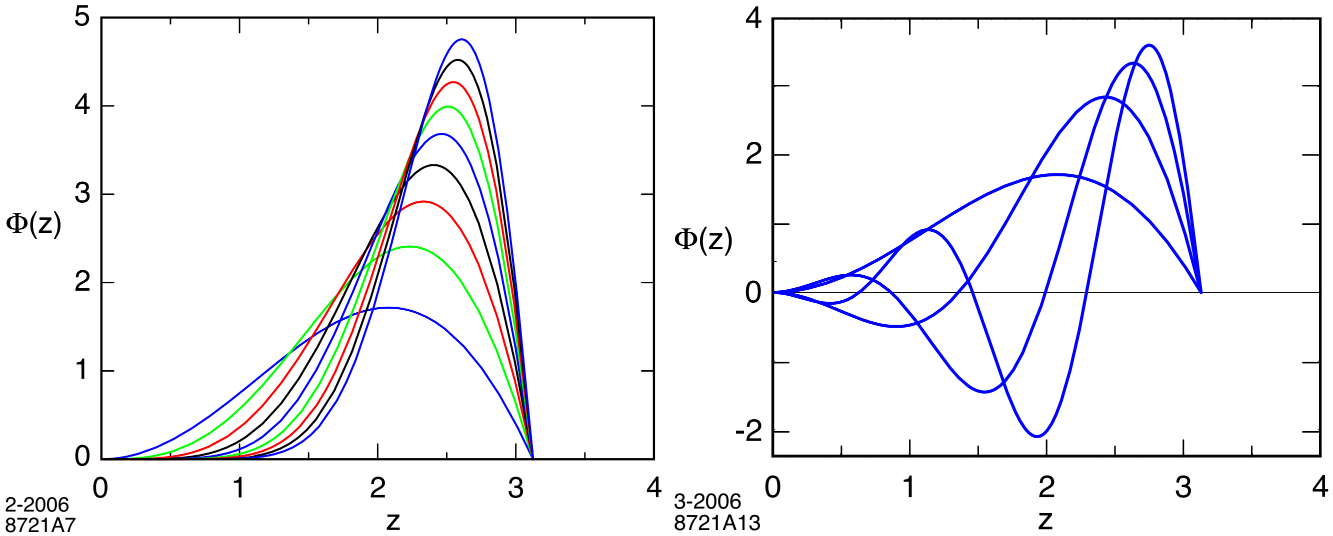


Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

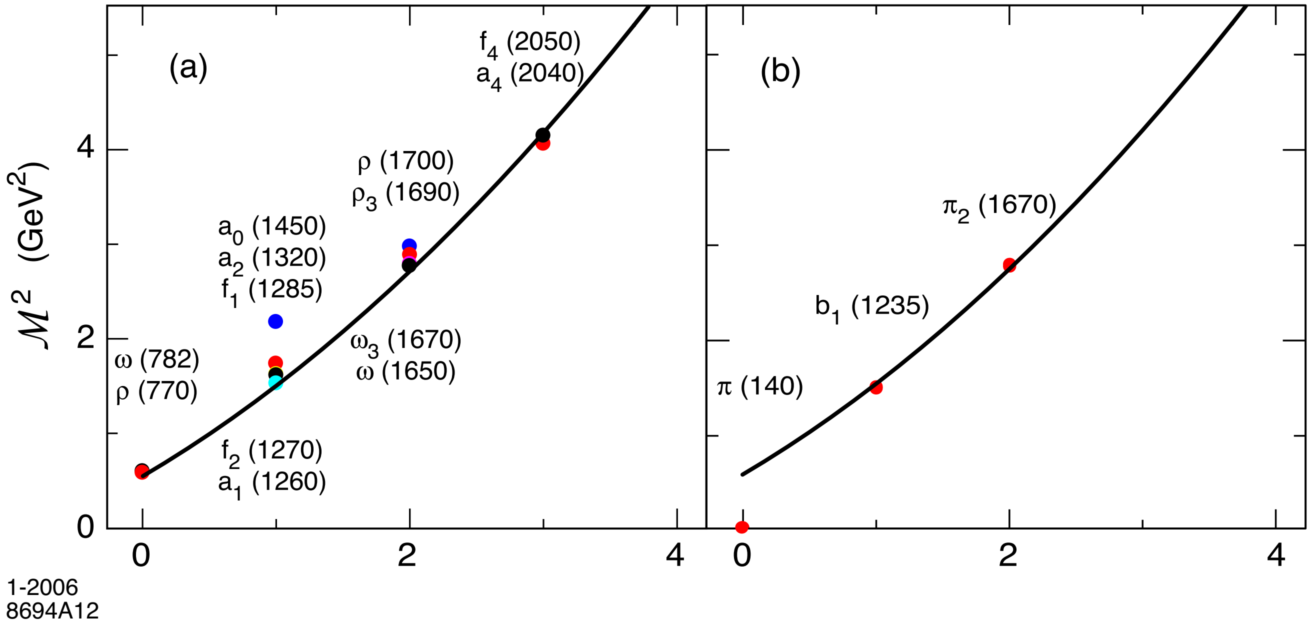


Fig: Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Baryon Spectrum

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i .$$

- Solve full 10-dim Dirac Eq., $\not{D}\hat{\Psi} = 0$, since baryons are charged under $SU(4) \sim SO(6)$. Baryon number conservation?

- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on compact space X with eigenvalues λ_κ:

$$\hat{\Psi}(x, z, y) = \sum_\kappa \Psi_\kappa(x, z) \eta_\kappa(y).$$

- From the 10-dim Dirac equation, $\not{D}\hat{\Psi} = 0$:

$$\left[z^2\, \partial_z^2 - d\, z\, \partial_z + z^2 \mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2}\left(\frac{d}{2} + 1\right) + (\lambda_\kappa + \mu) R\, \hat{\Gamma} \right] f(z) = 0,$$

$$i\not{D}_X \eta(y) = \lambda\, \eta(y),$$

where $\Psi(x, z) = e^{-iP\cdot x} f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and $\hat{\Gamma} u_\pm = \pm u_\pm$.

See: Muck and Viswanathan, hep-ph/9805945.

- Normalizable AdS baryon mode:

$$\Psi(x,z) = Ce^{-iP\cdot x} z^{\frac{d+1}{2}} \left[J_{(\mu+\lambda_\kappa)R-\frac{1}{2}}(z\mathcal{M})\, u_+(P) + J_{(\mu+\lambda_\kappa)R+\frac{1}{2}}(z\mathcal{M})\, u_-(P) \right].$$

  with $\Delta = \frac{d}{2} + |(\mu + \lambda_\kappa)R|$.

- For $d = 4$, $\hat{\Gamma} = \gamma_5$ and spinors $u_\pm(P)$ are defined along 4-dim coordinates.

- μ determined asymptotically by spectral comparison with orbital excitations in the boundary: $\mu = L/R$ and λ_κ are the eigenvalues of the Dirac equation on S^{d+1}:

$$\lambda_\kappa R = \pm \left(\kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2...$$

  See: Camporesi and Higuchi: gr-gc/9505009.

- Spin-$\frac{3}{2}$ Rarita-Schwinger eq. in AdS similar to spin-$\frac{1}{2}$ in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, Ψ_μ. See: Volovich, hep-th/9809009.

Introduction of Twist (Spin $\frac{1}{2}$ and $\frac{3}{2}$ AdS Modes)

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n} \sigma_i$.
- For a three quark state $\Delta \to \Delta - 3/2$. Change compensated in μ by the shift $L \to L - 1$ and $\Psi(z) \to z^{-\frac{1}{2}} \Psi(z)$.
- Three-quark baryon described by wave equation ($d = 4$, $\kappa = 0$)

$$\boxed{\left[z^2 \, \partial_z^2 - 3z \, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP \cdot x} z^2 \Big[J_{1+L}(z\mathcal{M}) \, u_+(P) + J_{2+L}(z\mathcal{M}) \, u_-(P) \Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^{\pm} = 0 \quad \Longrightarrow \quad$ <u>parallel Regge trajectories for baryons !</u>

$$\mathcal{M}_{\alpha,k}^{+} = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^{-} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

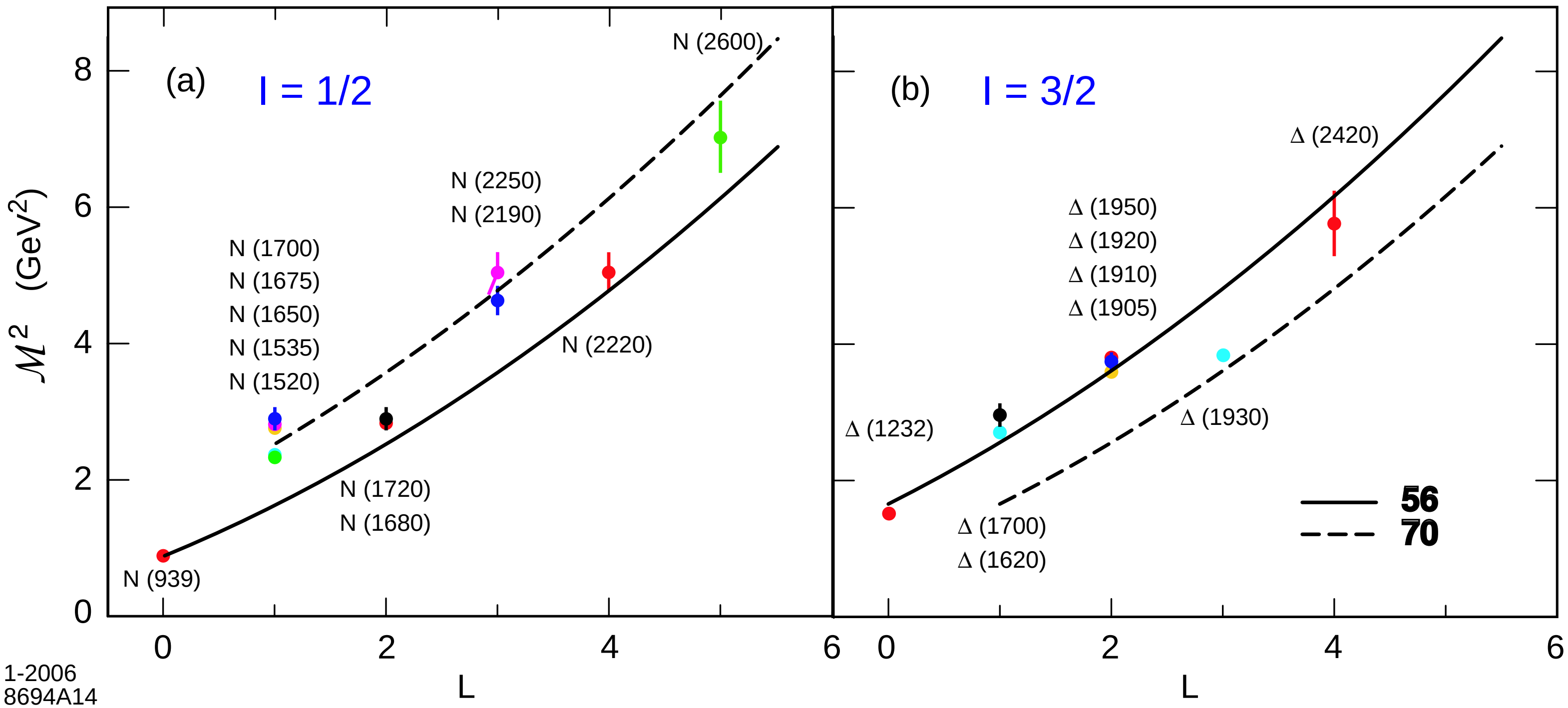


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Hadronic Form Factor in AdS/QCD

Mesons

- Hadronic matrix element for the electromagnetic current in AdS $x^\ell = (x^\mu, z)$:

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \left(\Phi_F^*(x,z) \partial_\ell \Phi_I(x,z) - \Phi_I(x,z) \partial_\ell \Phi_F^*(x,z) \right).$$

- Electromagnetic probe polarized along Minkowski coordinates

$$A_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q,z), \quad A_z = 0.$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0) = 1$, and as boundary limit, $z \longrightarrow 0$, the external current, $A^\mu = \epsilon^\mu e^{iQ \cdot x}$. Thus:

$$\lim_{Q \to 0} J(Q,z) = \lim_{z \to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q = 0$ and $z \longrightarrow 0$:

$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source:

$$F(Q^2)_{I \to F} = R^3 \int_0^\infty \frac{dz}{z^3} \, e^{3A(z)} \Phi_F(z) \, J(Q,z) \, \Phi_I(z)$$
$$\simeq R^3 \int_0^{z_o} \frac{dz}{z^3} \, \Phi_F(z) \, J(Q,z) \, \Phi_I(z).$$

- At large enough Q, the interaction occurs in the small-z conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

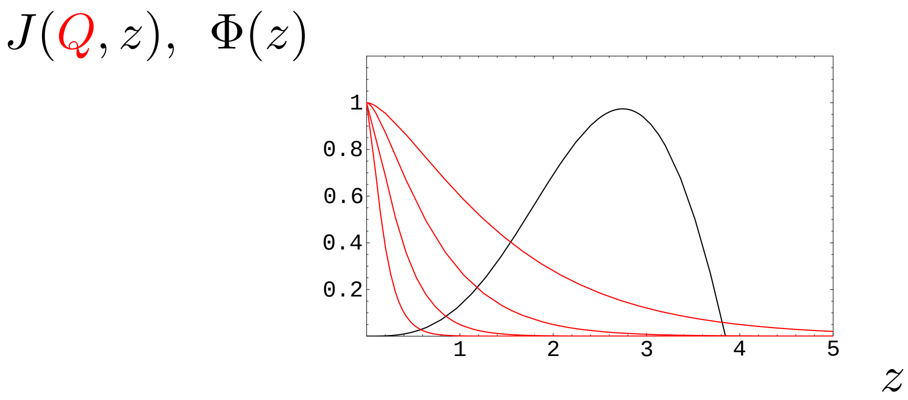


Fig: Suppression of external perturbations for large Q inside AdS.

- At small z, Φ scales as $\Phi \sim z^{\Delta}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\Delta-1}.$$

Hard scattering behavior for spinless constituents ($n = \Delta$) !

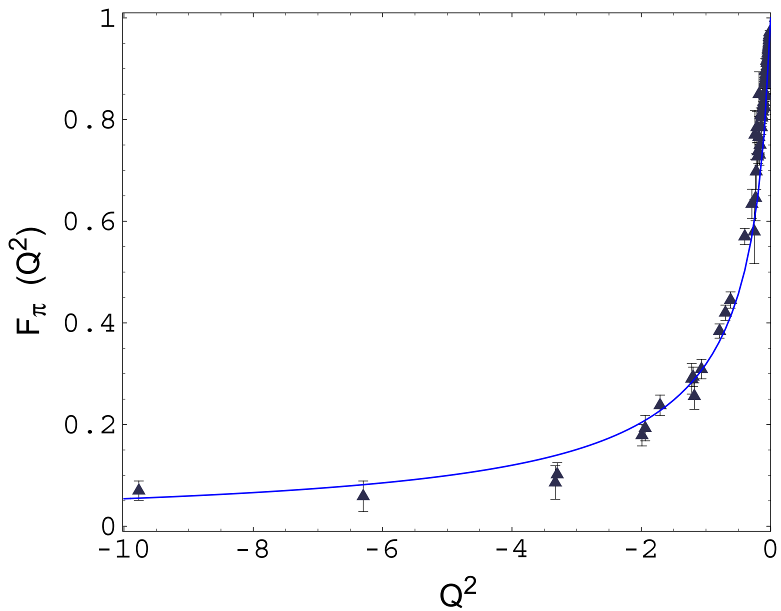
- For partons with spin σ there is an additional kinematical factor p^{σ} from the boost of the W.F.

- Spin carrying constituents $(\Delta \to \tau)$:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

where $\tau = \Delta_n - \sigma_n, \sigma_n = \sum_{i=1}^{n} \sigma_i$.

- The twist is equal to the number of partons, $\tau = n$.



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Baryons

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu \Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \quad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

- In the large P^+ limit

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}\,.$$

- Consider the spin non-flip form factors in the infinite wall approximation

$$\begin{aligned} F_+(Q^2) &= g_+ R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_+(z)|^2, \\ F_-(Q^2) &= g_- R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_-(z)|^2, \end{aligned}$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

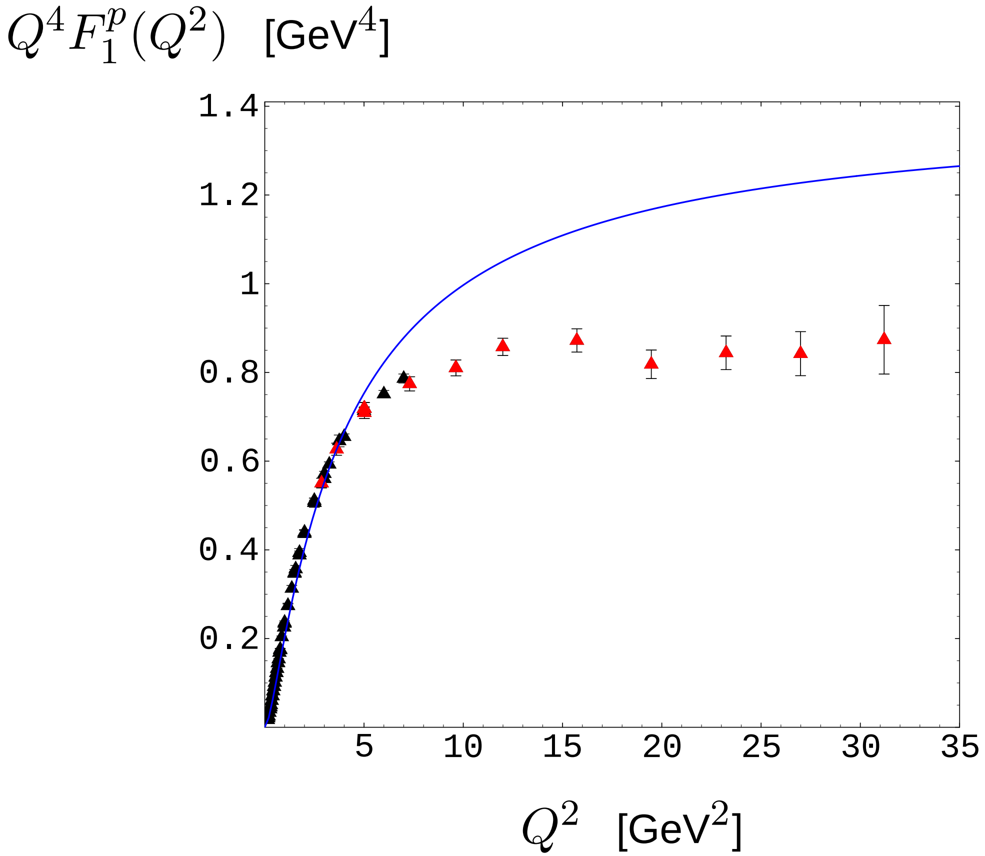
- For $SU(6)$ spin-flavor symmetry

$$\begin{aligned} F_1^p(Q^2) &= R^3 \int \frac{dz}{z^3} \, J(Q,z) |\psi_+(z)|^2, \\ F_1^n(Q^2) &= -\frac{1}{3} R^3 \int \frac{dz}{z^3} \, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right], \end{aligned}$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

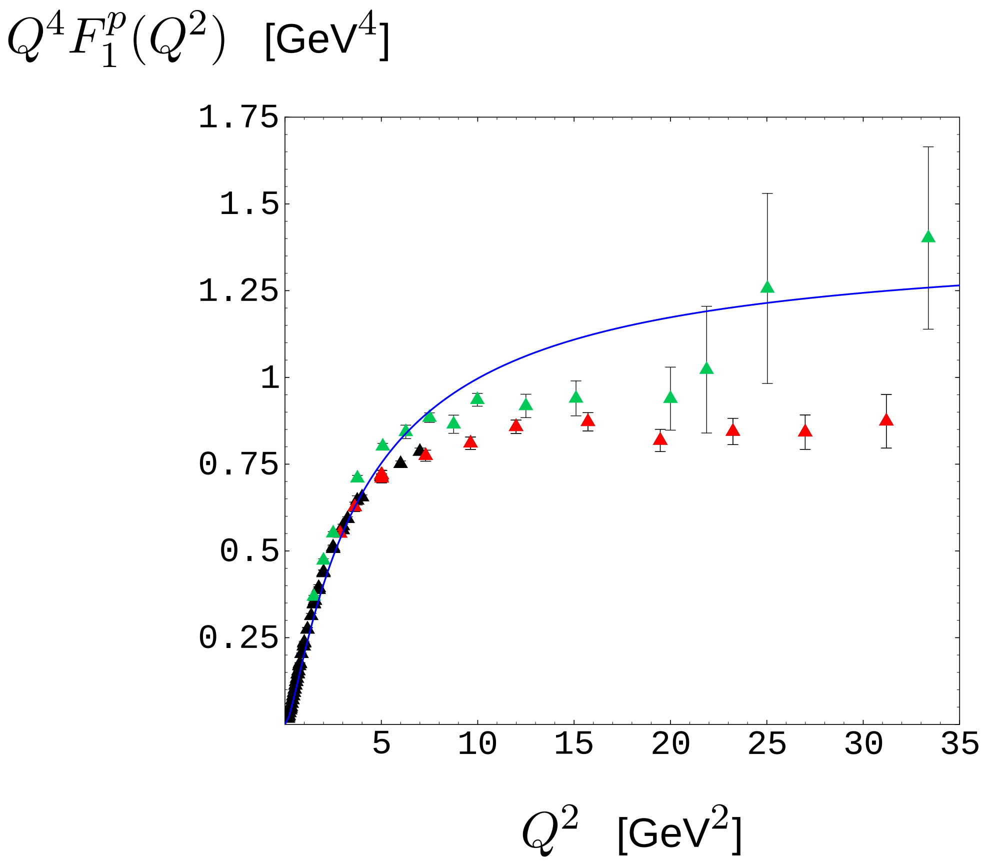
- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

Dirac Proton Form Factor F_1^p



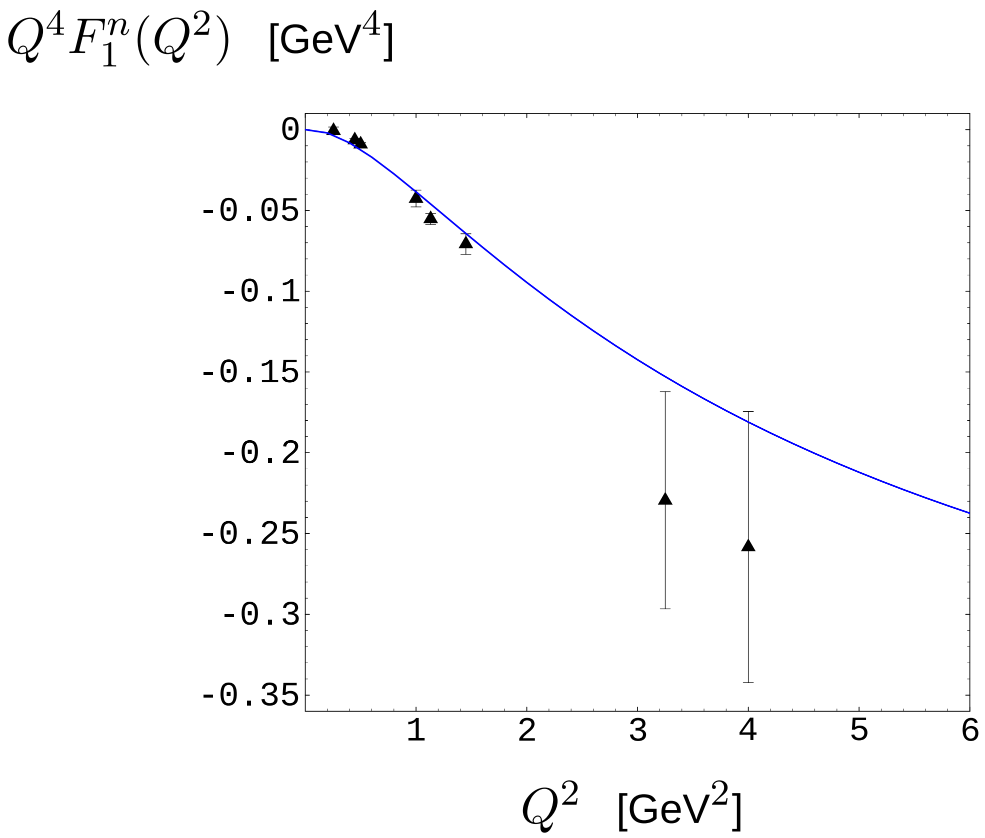
Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation. The analysis of the data is from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005). Red points correspondd to the Sill data: A. F. Sill *et al.*, Phys. Rev. D **48** (1993) 29.

Dirac Proton Form Factor F_1^p



Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation includding the data from Kirk (superimposed green points assuming $G_E^p = G_M^p$): P. N. Kirk *et al.*, Phys. Rev. D **8** (1973) 63.

Dirac Neutron Form Factor F_1^n



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation.

Holographic Model for QCD Light-Front Wavefunctions

SJB and GdT in preparation

- Drell-Yan-West form factor in the light-cone (two-parton state)

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

- Change the integration variable $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max} = \Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

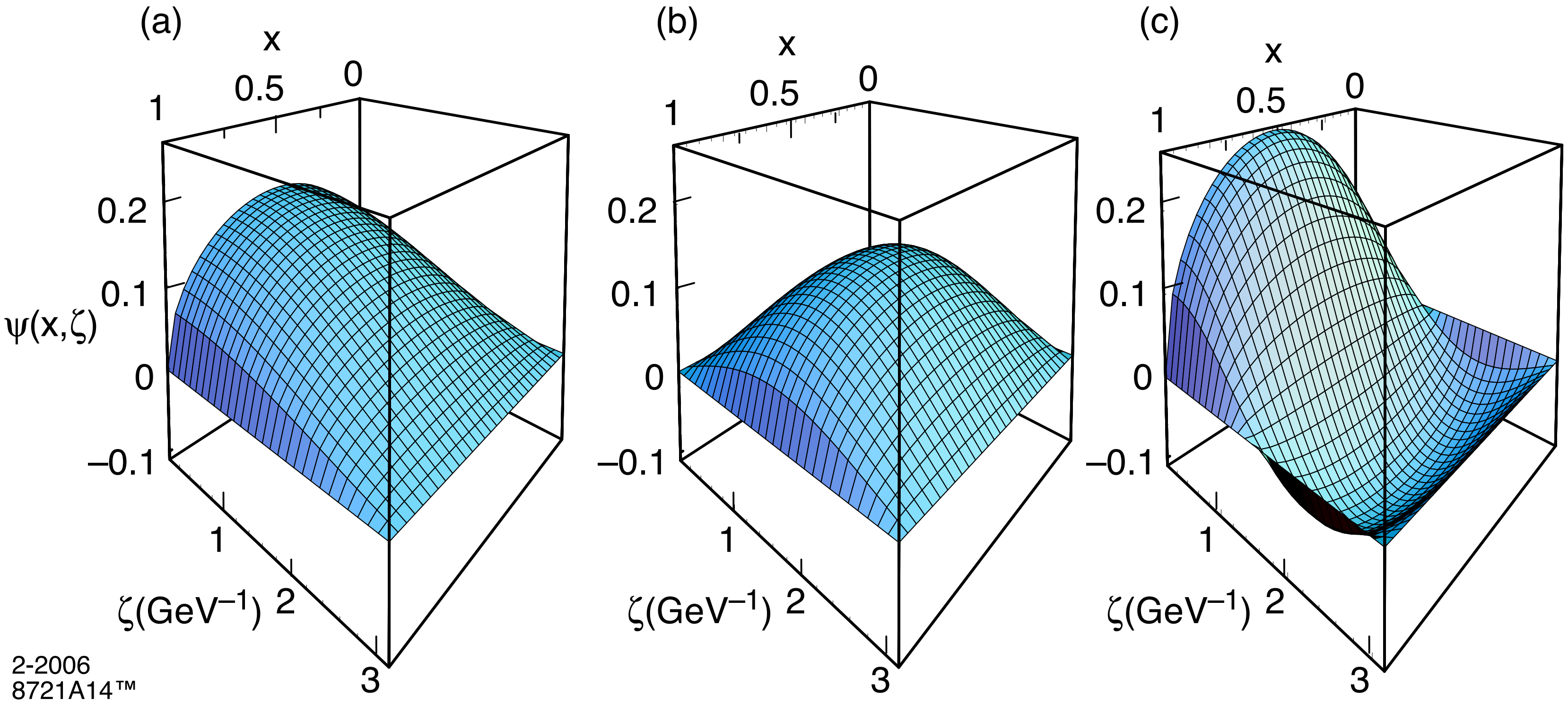
$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

  the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)}$$

  the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

Evaluation of QCD Matrix Elements: Example f_π

- Pion decay constant defined by the matrix element of EW current J_W^+:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ (1 - \gamma_5) \psi_d \right| \pi^- \right\rangle = i\sqrt{2} P^+ f_\pi,$$

with

$$\left|\pi^-\right\rangle = \left|d\overline{u}\right\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) \left|0\right\rangle.$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky, Phys. Rev. D **22**, 2157 (1980)

- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\rm QCD}}{8 J_1(\beta_{0,1})} = 83.4 \,{\rm Mev},$$

for $\Lambda_{\rm QCD} = 0.2$ GeV.

Experiment: $f_\pi = 92.4$ Mev.

Exact Holographic Mapping for Light-Front n-Parton State

- Define transverse position coordinates

$$x_i \vec{r}_{\perp i} = x_i \vec{R}_\perp + \vec{b}_{\perp i},$$

so that

$$\sum_{i=1}^{n} b_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i \vec{r}_{\perp i} = \vec{R}_\perp.$$

- DYW result for the form factor takes the convenient form:

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, |\widetilde{\psi}_n(x_j, \vec{b}_\perp)|^2 e^{i\vec{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}},$$

corresponding to a change of transverse momentum $x_j \vec{q}_\perp$ for each of the $n-1$ spectators.

- Struck constituent with momentum fraction x and $n-1$ spectators with total longitudinal momentum $1-x$.

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp).$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\widetilde{\psi}_n(x_j, \vec{b}_{\perp j})|^2.$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$\boxed{z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|}$$

- Our final result:

  hadronic QCD transverse density $\tilde{\rho}$ is determined by the string mode density $|\Phi|^2$ in AdS space!

$$\boxed{\tilde{\rho}(x,\zeta) = \frac{R^3}{2\pi} \frac{x}{1-x} e^{3A(\zeta)} \frac{|\Phi(\zeta)|^2}{\zeta^4}}$$

- The variable ζ, $0 \le \zeta \le \Lambda_{\rm QCD}^{-1}$, is related to the average transverse separation between spectator constituents, and it is also the holographic variable z, $\zeta = z$.

- For the two-particle case

$$\tilde{\rho}(x,\zeta) = \frac{1}{(1-x)^2} \left| \widetilde{\psi}\,(x,\zeta) \right|^2,$$

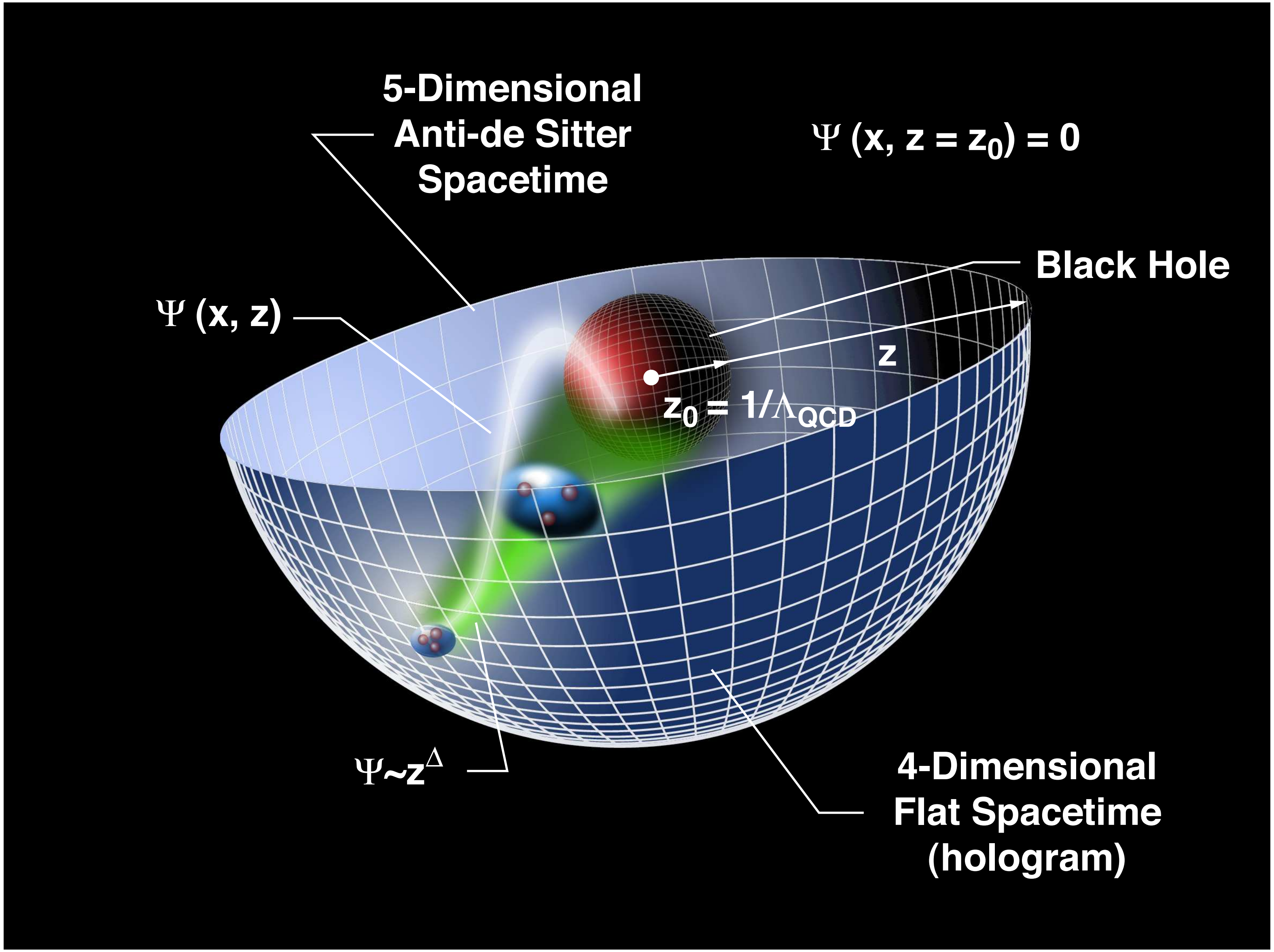
  and we recover our previous results

$$\left| \widetilde{\psi}(x,\zeta) \right|^2 \simeq \frac{R^3}{2\pi}\, x(1-x)\, \frac{|\Phi(\zeta)|^2}{\zeta^4} \theta\left(\zeta^2 \le \Lambda_{\rm QCD}^{-2}\right).$$

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Light-cone frame is the natural frame to establish the AdS/QCD holographic duality.
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- AdS modes dual to hadrons extrapolate to valence constituents at zero separation in the AdS boundary.
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Initial good approximation for description of the structure of hadronic form factors and other observables, but disagreement with data at very high Q^2 may indicate shortcomings of the hard wall approximation.

- Use of holographic light-front wave functions to compute hadronic matrix elements and other observables.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model, modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
- Precise mapping of string modes to partonic states. String modes inside AdS represent the probability amplitude for the distribution of quarks at a given scale.
- Exact holographic mapping for n-parton state determines effective QCD transverse charge density in terms of modes in AdS space.
- Holographic mapping allows deconstruction: express the eigenvalue problem in terms of 3+1 QCD degrees of freedom.



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