

Ph 205b Problem Set 6

1. a) Show that the LSZ reduction formula for $1 \rightarrow 1$ particle scattering is given by

$${}_{out}\langle p' s_1 | p s_2 \rangle_{in} = \left(\frac{1}{\sqrt{Z_2}} \right)^2 \lim_{\substack{p^2 \rightarrow m^2 \\ p^0 > 0}} \lim_{\substack{p'^2 \rightarrow m^2 \\ p'^0 > 0}} \int d^4x d^4y e^{ip' \cdot x} e^{-ip \cdot y} \langle \Omega | T \left\{ \left[\bar{u}^{s_1}(p') \frac{\not{p}' - m}{i} \psi(x) \right] \left[\bar{\psi}(y) \frac{\not{p} - m}{i} u^{s_2}(p) \right] \right\} | \Omega \rangle$$

b) Consider scattering of an electron by a static nucleus. The interaction is

$$S_{int} = e \int d^4x \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu^{ext}(x),$$

$$A_\mu^{ext} = \left(-\frac{Ze}{4\pi|\vec{x}|}, \vec{0} \right)$$

At lowest order in perturbation theory, show that

$${}_{out}\langle p' | p \rangle_{in} = -\frac{ie^2 Z}{2} \delta(E - E') \int d^3x \frac{e^{-i\vec{q} \cdot \vec{x}}}{|\vec{x}|} \bar{u}(p') \gamma^0 u(p)$$

where $\vec{q} = \vec{p} - \vec{p}'$, and we have suppressed spin indices.

2. Peskin and Schroeder problem 5.2 (Bhaba scattering).