Ph 205b Problem Set 5

1. a) Show that near $p^2 = m^2$,

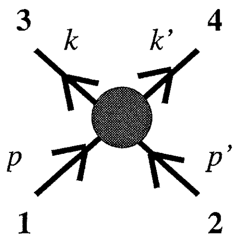
$$\int d^4x e^{ip\cdot x} \langle\Omega| T\left\{\psi(x)\bar{\psi}(0)\right\}|\Omega\rangle \to \frac{iZ(\not{p}+m)}{p^2 - m^2 + i\epsilon},$$

where $\langle\Omega|\,\psi(0)\,|p,s\rangle = \sqrt{Z}u^s(p)$. (Imagine that the photon has a small mass.)

b) What is the analogous formula for the time-ordered product of two photon fields?

2. In class, we derived the LSZ formula for ${}_{out}\langle \mathbf{p}_1\mathbf{p}_2|\mathbf{k}_1\mathbf{k}_2\rangle_{in}$ in scalar field theory. It related this matrix element to a time-ordered product of four fields. Derive the corresponding formulas for photons and fermions in QED.

3. Consider $2 \to 2$ scattering:



The Mandelstam variables s, t, u are defined by

$$\begin{aligned} s &= (p+p')^2 = (k+k')^2 \\ t &= (k-p)^2 = (k'-p')^2 \\ u &= (k'-p)^2 = (k-p')^2 \end{aligned}$$

a) Show that

$$s + t + u = \sum_{i=1}^{4} m_i^2,$$

where m_i is the mass of particle i.

b) In $e^-(p) + e^+(p') \to \mu^-(k) + \mu^+(k')$, show that

$$\frac{1}{4}\sum_{spins} |\mathcal{M}_{fi}|^2 = \frac{8e^4}{s^2}\left[\left(\frac{t}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right].$$