Ph 205b Problem Set 4

1. What are the momentum space Feynman rules for QED?

2. In quantum electrodynamics, calculate the renormalization constant Z_e using dimensional regularization with minimal subtraction.

3. Argue that for an integral of the form

$$\int \frac{d^n k}{(2\pi)^n} k^\alpha k^\beta k^\mu k^\nu f(k^2)$$

you can make the replacement

$$k^\alpha k^\beta k^\mu k^\nu \to \frac{1}{n(n+2)}[g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu}](k^2)^2$$

in the integrand. Where $g^{\mu\nu}$ are n dimensional spacetime metric, in flat spacetime it is simply $\eta^{\mu\nu}$.

4. A bonus problem
(You are not required to do this problem, but it's good to do more practice and to gain extra points :))

Consider a theory with Lagrangian density

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{int} \\
\mathcal{L}_0 &= \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi + \sum_{k=1}^{2} \bar{\psi}_k (i\partial\!\!\!/ - m_k)\psi_k \\
\mathcal{L}_{int} &= -g(\phi \bar{\psi}_1 \psi_2 + h.c.) - \frac{\lambda}{4!}(\phi^* \phi)^2
\end{aligned}$$

where ϕ is a complex scalar field and ψ_k's are Dirac fields.

a) What are the Feynman rules?

b) Calculate the renormalization Z factors for this theory. Use dimensional regularization with minimal subtraction. Work to order λ and order g^2.