Ph 205b Problem Set 3

1. Add to QED the following interaction terms:

$$\mathcal{L}' = \lambda_1 F_{\mu\nu} F^{\nu\lambda} F_\lambda{}^\mu + \lambda_2 \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi, \qquad \text{where } \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu].$$

a) What are the dimensions of the coupling constants λ_1 and λ_2?

b) Using our naive method, what are the momentum space Feynman rules associated with these interactions?

2. Show that

$$\frac{1}{a_1^{m_1} \cdots a_n^{m_n}} = \frac{\Gamma(M)}{\Gamma(m_1) \cdots \Gamma(m_n)} \int_0^1 dx_1 \, x_1^{m_1 - 1} \cdots \int_0^1 dx_n \, x_n^{m_n - 1} \frac{\delta(1 - \sum_{i=1}^n x_i)}{(a_1 x_1 + \cdots + a_n x_n)^M},$$

where $M = \sum_{i=1}^{n} m_i$, Γ is Gamma function $\Gamma(n) = (n-1)!$.

3. Consider the theory of two real scalar fields ϕ_1 and ϕ_2 with Lagrangian density

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{int}, \\ \mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 - m_1^2 \phi_1^2) + \frac{1}{2}(\partial_\mu \phi_2 \partial^\mu \phi_2 - m_2^2 \phi_2^2), \\ \mathcal{L}_{int} &= -g \phi_1 \phi_2^2 - \frac{\lambda_1}{4!} \phi_1^4 - \frac{\lambda_2}{4!} \phi_2^4 \, . \end{aligned}$$

a) What are the coordinate space and momentum space Feynman rules for this theory?

b) Using a momentum space cutoff Λ as a regulator, for particle 1, calculate $\langle \Omega | T\{\phi_1(x)\phi_1(y)\} | \Omega \rangle$. Drop terms that vanish as $\Lambda \to \infty$ and work to order λ_j (j=1,2) and order g^2. And extract its physical mass and Z_2.