Ph 205b Problem Set 1

1. Suppose that in quantizing the electromagnetic field we did not demand that the states $|b\rangle$ in the Hilbert space satisfy

$$\langle b|\partial_\mu A^\mu|b\rangle = 0.$$

Show that the Hilbert space would then contain negative norm states.

2. Show that for the free photon field quantized as in class

$$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{(-i)\eta_{\mu\nu}}{p^2+i\epsilon}.$$

3. Show that

$$\langle\Omega| = \lim_{T\to\infty(1-i\epsilon)} \langle 0|U(T,0)(e^{-iE_0T}\langle 0|\Omega\rangle)^{-1}.$$

4. Calculate (Notice: Do not use Wick's theorem. You will get points taken off if you use Wick's theorem.)

$$\left\langle 0 \middle| T\left\{\exp\left[-i\int dt \int d^3x \; \frac{\lambda}{4!}\; \phi_I^4(x)\right]\right\} \middle| 0 \right\rangle,$$

to order λ, using

$$\phi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p e^{ip\cdot x} + a_p^\dagger e^{-ip\cdot x}\right).$$

Express your answer in terms of the Feynman propagator $D_F(0)$.

5. Consider the 4 dimensional field theory for a real scalar field ϕ with Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi)(1+4\alpha\phi+4\alpha^2\phi^2) - \frac{1}{2}m^2\phi^2(1+2\alpha\phi+\alpha^2\phi^2).$$

a. What is the dimension of the parameter α?

b. Argue that this is actually a free (non-interacting) field theory in disguise. (Hint: it's an easy example of field redefinition.)