Ph 205b Problem Set 2

1. State and prove Wick's Theorem for anticommuting Fermi fields.

2. Show that the disconnected vacuum diagrams cancel out from the diagrammatic expansion of the two-point function

$$\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle$$

in $\lambda\phi^4$ theory, up to and including terms of order λ^2.

3. a) Show that Wick's Theorem can be written as

$$T\left[\phi_I(x_1)...\phi_I(x_n)\right] =: \exp\left[\frac{1}{2}\int d^4x d^4y D_F(x-y)\frac{\delta}{\delta\phi_I(x)}\frac{\delta}{\delta\phi_I(y)}\right]\phi_I(x_1)...\phi_I(x_n) :,$$

where the functional derivative $\frac{\delta}{\delta\phi_I(x)}$ satisfies

$$\frac{\delta\phi_I(y)}{\delta\phi_I(x)} = \delta^4(x-y).$$

b) Consider the following "interacting" field theory:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \rho(x)\phi(x)$$

where $\rho(x)$ is a c-number function with $\rho(x)\to 0$ as $|x|\to\infty$. Using the previous result, show that.

$$U(\infty,-\infty) = T\text{exp}\left[-i\int_{-\infty}^{\infty} dt H_I(t')\right]$$

is given by

$$U(\infty,-\infty) = \exp\left[-\frac{1}{2}\int d^4x d^4y \rho(x)\rho(y)D_F(x-y)\right] : \exp\left[-i\int d^4z\rho(z)\phi_I(z)\right] : .$$

c) Compute

$$P(n) = \frac{1}{n!}\int\frac{d^3k_1}{(2\pi)^3 2E_{k_1}}\cdots\frac{d^3k_n}{(2\pi)^3 2E_{k_n}}|\langle k_1...k_n|U(\infty,-\infty)|0\rangle|^2.$$

This is the probability of finding n particles in the far future starting with the vacuum in the far past. (Since $\rho(x)\to 0$ as $t\to\pm\infty$, the initial/final states coincice with those of the free field theory. Show that n is Poisson distributed:

$$P(n) = \frac{\lambda^n}{n!}e^{-\lambda},$$

with

$$\lambda = \int\frac{d^3k}{(2\pi)^3 2E_k}|\tilde{\rho}(\vec{k},E_k)|^2.$$

Here

$$\tilde{\rho}(\vec{k},E_k) = \int d^4x\rho(x)e^{ik\cdot x},$$

with $k^0 = E_k$.