1 Galileo's principle and light waves

It was common wisdom among scientists since the time of Galileo that velocity is a relative notion. That is, it makes no sense to say that the velocity of an object has some particular value; it only makes sense to say that the velocity of an object relative to something else has some particular value. In the physics parlance, velocity is "observer-dependent", or makes sense only in a particular "reference frame". [1]

The discovery of electromagnetic waves seemingly changed all this.[2] All familiar waves are waves in some medium: ocean waves are waves of the ocean water, sound is a wave of varying air pressure. Suppose this is true about the electromagnetic waves and in particular about light, and let us call this mysterious medium "aether". Now it seems that there is a special reference frame: the reference frame of the aether. And one might call the velocity of an object relative to aether its absolute velocity.

It was quickly realized that such a picture is incompatible with Galileo's principle that laws of nature look the same in all reference frames. Let c be the velocity of light in aether ($c \simeq 300000 km/sec$). Suppose I move in aether with velocity v and a light wave propagates in the same direction. It seems clear that with respect to me the velocity of light will be $c - v$. On the other hand, if the light wave propagates towards me, then I should find that its velocity is $c + v$. But this means that laws of light propagation look different in a moving frame. In particular, I can measure my speed with respect to the aether simply by measuring the light velocity in various directions.

Such experiments were indeed performed, with the goal of determining the absolute speed of the Earth's motion. But it was found that the speed

[1] In particular, in the reference frame of the Sun, the Earth is moving and the Sun is at rest, but in the reference frame of the Earth the Sun is moving and the Earth is at rest. You might say: "Wait a minute, I heard in my science class that it was a great discovery of Copernicus that it is the Earth which revolves around the Sun, not the other way around. Now you tell me both statements can be true, if one chooses the reference frame right"? Yes, this is correct. However, the reference frames of the Sun and the Earth are not on the same footing. The orbits of all planets look pretty simple in the reference frame of the Sun (they are pretty close to being circles), they are complicated in the reference frame of the Earth.

[2] Well, maybe I should have said that the discovery of light waves changed all this. It was realized in the beginning of the 19th century that light is a wave of some kind. But only towards the end of 19th century, thanks to the work of James Clerk Maxwell and Heinrich Hertz, was it realized that light is an electromagnetic wave.

of light is always c and independent of the direction.

2 Einstein's amazing discoveries: part 1

Einstein realized that if the relativity principle of Galileo always applies (and in particular the speed of light is always c), then some amazing consequences can be deduced by pure logic. Let me list some of these consequences and then explain how they can be deduced.

1. Two events which happen simultaneously for one observer may not happen simultaneously for another observer.

2. From the point of view of an observer "at rest", all processes in a moving lab (or moving ship, or moving anything) slow down. The closer the motion speed is to the speed of light c, the greater is this slowdown. If the speed is very close to the speed of light, everything happening in the moving lab looks "frozen" to the observer "at rest".

3. All objects in motion shrink in the direction of their motion. If the motion speed approaches c, the size of the object in the direction of motion ("longitudinal size") becomes very small, i.e. a very fast-moving object looks like a thin pancake.

Let me begin by explaining (1). Imagine a long train in the middle of which there is a light bulb, and at the ends of which there are two photoelements (devices which can register when light hits them). Imagine that the train is moving to the right. If the light bulb goes on, then from the point of view of the observer on the train the light will reach both photoelements at the same time, since the speed of light is the same in both directions, and the light has equal distance to go. But from the point of view of the observer "at rest", the photoelement at the back of the train will go off first. Indeed, while the light propagates towards the back of the train (i.e. to the left), the back of the train is moving to the right, so clearly the light will need to travel less than half the length of the train. On the other hand, light that propagates to the the front of the train (i.e. to the right) needs to travel more than half the length of the train, because during its travel the front end of the train moves to the right. (Draw a picture of the train at various times if you got confused; this should help.)

The conclusion is that the two observers disagree whether the two photoelements went off at the same time or not.

Now let me explain statement (2). Imagine that on the same train we have installed a pair of mirrors on the ceiling and the floor, one right above the other. Imagine a light pulse going back and forth between the two mirrors. How long does it take for the light pulse to travel back and forth once? The observer on the train says that it takes $2h/c$ seconds, where h is the distance between the floor and the ceiling. The observer "at rest" will get a different number. From his point of view, since the train is moving while the light pulse is travelling, the trajectory of the light pulse is not vertical: it looks like a letter "V" turned upside-down. Clearly, the length of this trajectory is larger than $2h$, therefore a roundtrip will take more than $2h/c$ seconds.

Let us actually do this computation and see how great this difference between the times measured by the two observers is. If the speed of the train is v and the roundtrip takes time t from the point of view of an observer "at rest", then the horizonal distance that light travels during its roundtrip is vt. The total distance that the light pulse travels (the length of two legs of "V") must be ct, thus each leg of "V" has length $ct/2$. Pythagoras theorem then says

$$h^2 + (vt/2)^2 = (ct/2)^2.$$

Solving this with respect to t we find

$$t = \frac{2h/c}{\sqrt{1 - \frac{v^2}{c^2}}} > 2h/c.$$

One can regard this device (a pair of mirrors and a light pulse going back and forth) as a primitive clock. Then one can formulate our result as follows: from the point of view of the observer "at rest" such a clock slows down when it is in motion. But then Galileo's principle forces ALL moving clocks to slow down by the same amount! Indeed, if they did not all slow down at the same rate, by comparing various clocks the observer on the train could discover that he is moving without ever looking out of the window. And this is not possible by Galileo's principle.

Since all clocks slow down at the same rate, one can say that *time itself slows down on a moving train.* More precisely, if the moving observer thinks that a time t_0 has passed, the observer "at rest" thinks that time that passed is

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

To explain statement (3) consider three identical spaceships traveling in a file formation. Suppose that the distance between ships 1 and 2 is the same as distance between 2 and 3 and equal to L. The commander of this fleet is in the middle spaceship. He decides to accelerate a little and at time $t = 0$ sends radio signals to ships 1 and 3 to turn on the rocket engines for a short time. He reckons that the signals will reach both spaceships after time L/c, so he waits this time after sending the order and turns on his own rocket engine. He thinks that then all three spaceships will start accelerating at the same time and therefore the distance between them will not change. But the observer at rest will see things differently. He sees the that the radio signal first reaches spaceship 3, and only then spaceship 1. This happens because the radio signal which travels back has to travel less than L, while the signal which travels forward has to travel more than L. Thus according to the observer "at rest" the sequence of events is this: first ship 3 starts accelerating, then ship 2, and only then ship 1. But then in the beginning the distance between ships 2 and 3 will shrink! And so will the distance between 1 and 3. That is, after the completion of the order all 3 ships will be closer together than in the beginning. The observer "at rest" can prove it: he filmed the whole sequence of events on tape.

But how can this be? After all, if the distance between the ships gets small enough, they might bump into each other! Surely all observers must agree on whether the ships bumped into each other or not.

According to Einstein, the resolution of the paradox is this: as ships accelerate, their longitudinal size gets smaller, from the point of view of the observer "at rest". Although the distances between all 3 ships will get smaller by some factor after the acceleration is over, their lengths also get smaller by the same factor. So there will be no collision, according to all observers. And this is what the observer "at rest" will see and record on tape.

A careful computation shows that the length changes as follows: if l_0 is the length of the ship at rest, then its "moving length" is

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

Note the same square-root factor as the formula for time slowdown. It appears in many formulas in relativity theory.

By the way, not only the length of the ship will shrink by this factor. Everything on the ship, including the astronauts, will shrink by the same factor. Otherwise by measuring lengths of various objects the astronauts would be able to infer their absolute velocity, which would contradict Galileo's principle.