Homework 8

1. (a) (10 pts) Open strings can be related to closed strings using the "doubling trick". For example, when Neumann boundary conditions are used at both ends (NN), one can represent a scalar field on an interval $(0, \pi)$ by a scalar field on the interval $(0, 2\pi)$ satisfying the usual periodic boundary condition $X^\mu(\sigma + 2\pi) = X^\mu(\sigma)$ as well a supplementary condition $X^\mu(2\pi - \sigma) = X^\mu(\sigma)$. Write down similar conditions for the DD, DN, and ND boundary conditions, where D stands for Dirichlet. Write down the mode expansions for all these cases and show that in the DN and ND sectors the vacuum (Casimir) energy is shifted up compared to the DD and NN sectors.

(b) (10 pts) The doubling trick can be used for fermionic fields as well. For example, with NN boundary conditions one has a supplementary condition $\psi_+(2\pi - \sigma) = \psi_-(\sigma)$, along with either the Ramond condition $\psi_+(2\pi + \sigma) = \psi_+(\sigma)$ or the Neveu-Schwarz condition $\psi_+(2\pi + \sigma) = -\psi_+(\sigma)$. Write down similar supplementary conditions for the DD, DN, and ND sectors. Also write down mode expansions for all these cases and determine for which cases there are fermionic zero modes.

(c) (10 pts) Worlds-sheet parity acts on fields on the open-string world-sheet by

$$\Omega : X^\mu(\sigma) \mapsto X^\mu(\pi - \sigma), \quad \psi_\pm(\sigma) \mapsto \psi_\mp(\pi - \sigma).$$

Determine the action of Ω on oscillators in the DD and NN sectors (including both Ramond and Neveu-Schwarz cases for fermions).