Homework 4

1. (10 pts) Let B be a constant 2-form on a d-dimensional torus T^d, and G be a constant unit metric on T^d. Assume that the torus is given as $\mathbb{R}^d/(2\pi\mathbb{Z})^d$, so that shifting $B \to B + \alpha' N$, where N is a skew-symmetric integer-valued matrix, is a symmetry. As explained in class (see also Polchinski section 8.4), the data (G, B) define an even self-dual lattice $\Gamma \subset \mathbb{R}^{d,d}$, and N defines an automorphism of this lattice. Let us set $G = 1$ and $B = 0$. Show that in this case an automorphism of the lattice Γ can be identified with an element of $O(d, d, \mathbb{Z})$, and determine this element for the automorphism corresponding to the skew-symmetric matrix N.

2. (10 pts) Problem 8.5 in Polchinski.