

Homework 5

1. (10 pts) As explained in class, the state-operator correspondence for the bc ghost system is a bit unusual. Namely, the identity operator maps to the state $b_{-1}|\downarrow\rangle$ which has ghost number $-3/2$. Determine the states corresponding to the ghost field $c(z)$ and the anti-ghost field $b(z)$. Which operators correspond to the ghost vacuum states $|\downarrow\rangle$ and $|\uparrow\rangle$?

2. (10 pts) As explained in class, in the old covariant quantization of the open string, the state $|0, p\rangle$ is physical when $p^2 = a$, where a is the normal-ordering constant in the equation $(L_0 - a)|\Phi\rangle = 0$, while the first excited state $\epsilon_\mu \alpha_{-1}^\mu |0, p\rangle$ is physical if $p^2 = a - 1$ and $\epsilon_\mu p^\mu = 0$. Thus for $a = 1$ the first excited level describes a massless vector particle, while for $a < 1$ it describes a massive vector particle. (And the case $a > 1$ is ruled out because one finds negative-norm states). Extend the analysis to the next excited level and determine the spectrum. In the process you should also determine the range of a and D (dimension of space-time) for which the norm of all physical states is non-negative.