Homework 4

A chiral $U(1)$ current is a conformal primary $j(z)$ of dimension $(1,0)$ and the following Operator Product with itself:

$$j(z)j(w) = \frac{k}{(z-w)^2} + \ldots$$

The number k is known as the level of the current. A field $\phi(z)$ is called a primary with respect to a $U(1)$ current if its OPE with $j(z)$ looks as follows:

$$j(z)\phi(w) = \frac{q_\phi \; \phi(w)}{z-w} + \ldots.$$

The number q_ϕ is called the $U(1)$ charge of ϕ. (Do not confuse the notion of a primary with respect to a $U(1)$ current with a primary defined using the stress-energy tensor: they are different notions).

1. (10 pts) Let $\phi(z)$ be a primary with respect to a $U(1)$ current j. Determine the commutation relations between the modes of ϕ and the modes of j. Reformulate the condition of being a primary with respect to a $U(1)$ current in terms of the state $|\phi\rangle$ corresponding to the field ϕ.

2. (10 pts) Consider the CFT of a free scalar field X. Set $\alpha' = 1$ for simplicity. This CFT has a chiral $U(1)$ current $j = i\partial X$ and an anti-chiral $U(1)$ current $\bar{j} = i\bar{\partial} X$. Show that the only fields in this theory which are primary with respect to both j and $\bar{j}$ are vertex operators e^{ipX}. Determine the corresponding $U(1)$ charges.