Week 3 (due Jan. 27)

1. Probably the simplest quantum system exhibiting an anomaly of a global symmetry is a system of N real fermions ψ^α with an action

$$S = \int i g_{\alpha\beta} \psi^\alpha \frac{d\psi^\beta}{dt} dt,$$

where $g_{\alpha\beta}$ is a real symmetric non-degenerate matrix. On the classical level, the fermions satisfy $\{\psi^\alpha, \psi^\beta\} = 0$, while on the quantum level they satisfy

$$\{\psi^\alpha, \psi^\beta\} = g^{\alpha\beta},$$

where g with upper indices is the inverse of g with lower indices.

Show that for N even the natural Hilbert space for this system is a spinor representation of $O(N)$, i.e. the sum of a left-handed and a right-handed spinor of $SO(2N)$. Show that for N odd, the most natural Hilbert space is a representation of $SO(N)$ only, but not of $O(N)$. You may consider only the cases $N = 1, 2, 3$, for simplicity.

2. Problem 30.3 in Schwartz.