Week 6 (due May 11)

Reading: Terning chapter 8.

1. Problem 2 in Chapter 8 of Terning's book.

2. Consider $N = 1$ SUSY gauge theory with gauge group $SU(N_c)$ and a chiral superfield Φ in the adjoint representation.

(a) If the superpotential $W(\Phi)$ vanishes, show that the theory has the space of supersymmetric vacua of dimension $N_c - 1$. More precisely, show that the moduli space of vacua $\mathcal{M}$ can be identified with a quotient of a hyperplane in $\mathbb{C}^{N_c}$ with an equation $x_1 + \ldots + x_{N_c} = 0$ by the group of permutations of $x_1, \ldots, x_{N_c}$.

(b) What is the low-energy gauge group at a generic vacuum? Describe the low-energy effective theory around a generic vacuum, on the classical level. More precisely describe the chiral superfields and the F-terms (the D-terms are not important).

(c) Show that the theory with zero superpotential has a non-anomalous $U(1)_R$ symmetry. Show also that the theory is asymptotically free and strongly coupled in the infrared.

(d) In part (b), you should have found that in the low-energy effective theory the superpotential is zero on the classical level. Using the results of (c), show that it is zero even non-perturbatively.

(e) Now consider adding a superpotential $W(\Phi) = \frac{1}{2} m \mathrm{Tr} \Phi^2$. Show that the theory has a unique vacuum, with a gaugino conodensate. Use holomorphy to determine the dependence of the gaugino condensate on m and the string coupling scale Λ of the theory.