Week 9 (due March 11)

1. Compute the Chern-Simons 5-form and 7-form.

2. Consider a gauge theory on a Riemannian 3-manifold M with gauge group $U(N)$ and an action

$$\int_M \mathrm{Tr}\,(F \wedge *F) + m \int_M CS(\omega),$$

where $F = d\omega + \frac{1}{2}[\omega, \omega]$ is the curvature, $CS(\omega)$ is the Chern-Simons 3-form, and m is a parameter. Derive the equations of motion corresponding to this action. Show that in the limit $m \to \infty$ they reduce to $F = 0$.

3. Consider a gauge theory on a manifold M which contains, besides the gauge field ω on a principal G-bundle P, a section s of an associated complex vector bundle E. We also assume that $\mathrm{Ad}(P)$ and E have invariant scalar products (Euclidean and Hermitian, respectively). Take the action to be

$$\frac{1}{2e^2} \int_M \langle F, \wedge * F \rangle + \int_M \langle \nabla s, \wedge * \nabla s \rangle,$$

where e^2 is a parameter. Derive the equations of motion corresponding to this action.