Week 6 (due May 13)

1. Consider the space of complex square matrices of size $n \times n$. This is a complex vector space of dimension n^2. Consider a symplectic form on it:

$$\omega = \frac{i}{2} \mathrm{Tr}\, dZ \wedge dZ^\dagger,$$

where Z is the matrix. The symplectic form is obviously invariant under the $U(n)$ action $Z \mapsto UZU^{-1}$.

(a) Compute the moment map for this action.

(b) Show that the symplectic quotient of the space of matrices by the $U(n)$ action (at zero "level", i.e. we require the moment map to be identically zero) is isomorphic to the quotient of $\mathbb{C}^n$ (with its standard symplectic form) by the permutation group S_n which permutes all coordinates.

2. One might think that compact phase space is rather exotic. To dispel this notion, consider a nonrelativistic charged spinless particle of mass m and charge e moving on the xy plane and subject to a constant magnetic field B in the z direction. Further, we will assume that x and y directions are periodically identified, with periods L_x and L_y.

(a) Show that in the limit $m \to \infty$ the phase space is a torus; compute the corresponding symplectic form.

(b) Determine the conditions on B which ensure the existence of the pre-quantum line bundle.