Fall quarter, week 4 (due Oct. 31)

1. Compute all the cohomology groups of the Klein bottle (with integer coefficients). You can use either the cell complex or the universal coefficient theorem (since in the previous homework you already computed the homology groups of the Klein bottle). Also compute the cohomology of the Klein bottle with coefficients in $\mathbb{Z}_2$.

2. $Ext(A,B)$ was defined in class using a presentation of A as a quotient of two free abelian groups. Show that $Ext(A,\mathbb{R})$ and $Ext(A,U(1))$ vanish for any finitely-generated abelian group A.

3. The Euler characteristic of a space X is $\chi(X) = \sum_n (-1)^n b_n(X)$, where $b_n(X)$ are Betti numbers. Use the Kunneth formula to show that $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$.

4. Consider the short exact sequence $0 \to \mathbb{Z}_2 \to \mathbb{Z}_4 \to \mathbb{Z}_2 \to 0$, where the second map is multiplication by 2. For any topological space X it gives rise to Bockstein homomorphisms $\beta_n : H^n(X,\mathbb{Z}_2) \to H^{n+1}(X,\mathbb{Z}_2)$. Show that $\beta_{n+1} \circ \beta_n = 0$ for all n. (This operation on mod-2 cohomology classes is also known as the first Steenrod square Sq^1).