Week 9 (due March 12)

Reading: Srednicki, sections 51, 52.

1. (20pt) Problem 52.3 in Srednicki.

2. Consider a theory of a spinor field Ψ with the Lagrangian

$$L = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi + g\bar{\Psi}\Psi\phi(x),$$

where $\phi(x)$ is some function of x (not a quantum field). One refers to $\phi(x)$ as an external field. If g is small, we can treat the second term in the Lagrangian as a perturbation. This theory describes scattering of "electrons" and "positrons" by the external potential $\phi(x)$ (which in general is both space- and time-dependent).

(a) Deduce the Feynman rules for computing the scattering in this theory (in momentum space). Show that the resulting vertex is proportional to the Fourier transform of $\phi(x)$ evaluated at the 4-momentum $q = p' - p$, where p' and p are final and initial 4-momenta of the "electron".

(b) Now suppose the function $\phi(x)$ is independent of time. Then its Fourier transform is proportional to $\delta(E' - E)$, where E' and E are final and initial electron energies. That is, the scattering amplitude can be written as

$$2\pi\delta(E' - E)\, T_{fi}.$$

Show that the scattering cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{m}{16\pi^2 E}|T_{fi}|^2.$$

(c) Now assume that $\phi(t, \mathbf{x}) = a/r$, where $r^2 = \mathbf{x}^2$. (This is a toy model for relativistic scattering in the Coulomb field). Compute the cross-section as a function of energy and scattering angle.