Week 6 (due Feb. 20)

Reading: Srednicki, section 41, and any book which discusses nonrelativistic limit of the Dirac equation (e.g. Bjorken and Drell or Landau-Lifshits vol.4).

1. (a) Consider the theory of a real scalar field ϕ with a Lagrangian

$$L = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4.$$

Show that in the nonrelativistic limit (i.e. the limit where the momentum of all particles is much smaller than m) this Lagrangian describes, at leading order in p/m expansion, bosonic particles interacting via the 2-body potential $V(x_i - x_j) = a\delta^3(x_i - x_j)$. Express the coefficient a in terms of λ.

(b) At higher orders in $1/m$ expansion the interaction becomes more complicated. Find the first relativistic correction to the result of part (a).

(c) Consider the theory of a Dirac spinor field Ψ with the Lagrangian

$$L = \bar{\Psi}\left(i\partial\!\!\!/ - m\right)\Psi - \frac{1}{2}\lambda(\bar{\Psi}\Psi)^2.$$

Show that in the nonrelativistic limit this Lagrangian describes spin-1/2 particles interacting via a delta-function potential independent of the spin. Express the coefficient in front of the delta-function in terms of λ.

(d) The same as (c), but for the interaction Lagrangian

$$L_{int} = -\frac{1}{2}\lambda(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi).$$

(e) Invent a Lagrangian for the Dirac field which in the nonrelativistic limit gives rise to spin-dependent interaction potential.