Week 5 (due Feb. 13)

Reading: Srednicki, sections 39, 23, 40.
1. (30pts) Problem 39.4.
2. Problem 40.1.
3. Consider a theory of \(N\) Weyl fermions \(\chi^i, i = 1, \ldots, N\). The most general quadratic Hermitian Lorenz-invariant Lagrangian of first order in derivatives is

\[
L = i\chi^i_\dagger \sigma^\mu \partial_\mu \chi^i - m_{ij} \chi^i \chi^j - m^{*}_{ji} \chi^{i\dagger} \chi^{j\dagger}.
\]

(a) Show that without loss of generality the matrix \(m_{ij}\) can be taken to be symmetric. Show that this Lagrangian describes a theory of \(N\) independent massive Majorana particles. What are their masses? (Hint: an arbitrary complex symmetric matrix \(M\) can be “diagonalized”, i.e. one can find a unitary matrix \(U\) such that \(U^tMU = D\) is diagonal and the diagonal entries are nonnegative real numbers.)

(b) If \(N\) is even, one can arbitrarily pair up Weyl spinors \(\chi^i\) into \(k = N/2\) Dirac spinors \(\Psi^p, p = 1, \ldots, k\). Rewrite the above Lagrangian in terms of \(\Psi\). Note that the mass terms in this new Lagrangian are of two kinds: Dirac-type terms of the form \(A_{pq} \bar{\Psi}^p \Psi^q\) and Majorana-type mass terms.

(c) What are the continuous symmetries of this theory for generic \(m_{ij}\)? Show that for generic \(m_{ij}\) the theory is invariant under suitably defined parity and time-reversal symmetries. How do these symmetry transformations act on the fields \(\chi^i\)?