Week 5 (due Nov. 7)

Reading: Srednicki, sections 6 and 7.

1. The transition amplitude in nonrelativistic Quantum Mechanics is defined by

$$K(q', q; T) = \langle q' | e^{-iHT} | q \rangle.$$

Here H is the usual Hamiltonian, i.e.

$$H = \frac{\hat{p}^2}{2m} + V(q).$$

(a) Compute $K(q', q; T)$ for the free particle ($V = 0$) by inserting identity operators in the form

$$1 = \int dp |p\rangle \langle p|,$$

and then evaluating the resulting matrix elements and integrals over p.

(b) On the other hand, one can consider the second-quantized version of the same system and the corresponding 2-point Green's function

$$G(q', q; T) = \langle 0 | \Psi(T, q') \Psi^\dagger(0, q) | 0 \rangle$$

Show that for any potential $V(q)$ one has $G(q', q; T) = K(q', q; T)$. Verify this in the special case $V = 0$ by directly evaluating $G(q', q; T)$ using the known Fourier-expansion of Ψ and $\Psi^\dagger$ in terms of creation-annihilation operators and then comparing with the results of part (a).

(c) The path-integral representation for $K(q', q; T)$ is

$$K(q', q; T) = \int Dq(t) \exp(iS),$$

where

$$S = \int dt \left(\frac{m}{2} \dot{q}^2 - V(q) \right).$$

In more detail, the path-integral is defined as the limit

$$\lim_{N \to \infty} F(\epsilon)^N \int dq_1 \ldots q_{N-1} \exp \left[i\epsilon \sum_{i=0}^{N-1} \left(\frac{m}{2} (q_{i+1} - q_i)^2 / \epsilon^2 - V(q_i) \right) \right],$$

where $\epsilon = T/N$, $q_0 = q$, $q_N = q'$, and the function $F(\epsilon)$ should be chosen so that in the limit $N \to \infty$ one gets the correct expression for $K(q', q; T)$.

Determine $F(\epsilon)$ by evaluating the integral in the special case $V = 0$ and comparing with the results of part (a).

(d) Consider now the case $V(q) = -fq$. This corresponds to a particle which is acted upon by a constant force f. Find $K(q', q; T)$ by evaluating the path-integral and using the function $F(\epsilon)$ found in part (c).