Week 4 (due Oct. 31)

Reading: Srednicki, sections 4 and 5.

1. Compute the commutator function $[\phi(x), \phi(0)]$ for the free real scalar field ϕ with zero mass ($m = 0$). Hint: by rotational invariance, you may assume that the spatial part of x is along the x^1 axis. The integral over k_2 and k_3 is easily computed, if we recall that

$$\frac{d^3k}{2\omega_k} = d^4k\, \delta(-k^2)\theta(k^0).$$

The remaining integral over k^0 and k^1 is most easily evaluated in the "light-cone coordinates" $k_+ = k^0 - k^1$ and $k_- = k^0 - k^1$.

2. Let ϕ be as in problem 1. Compute the vacuum expectation value

$$< 0|\phi(x)\phi(0)|0 > .$$

Hint: be careful, this is a distribution, not a function. Use the same method as in problem 1.

3. One may consider second-quantized nonrelaivistic quantum mechanics as a quantum field theory. If the particles are noninteracting and the external potential vanishes, its action is given by

$$S = \int dt\, d^3x \left(i\Psi^\dagger \partial_t \Psi - \frac{1}{2m} \partial_j \Psi^\dagger \partial_j \Psi \right).$$

Here one can regard Ψ either as a classical field, or as a field operator.

(a) The above action has several obvious symmetries: translation in space and time, rotations, and phase symmetry:

$$\Psi \mapsto e^{i\alpha}\Psi.$$

Using Noether theorem, find the corresponding conserved currents and charges (charge is the integral of the time component of the current over space).

(b) Less obviously, the action is also invariant under "nonrelativistic boost" transformations

$$x^i \mapsto x^i - v^i t, \quad t \mapsto t,$$

provided the field Ψ is also transformed appropriately. Here v^i is the constant velicity vector parametrizing the boost. Find the transformation law for Ψ that achieves this, and using Noether theorem deduce the corresponding conserved current. (Hint: the answer to the first part of the question is contained, for example, in Landau and Lifshits, "Quantum mechanics", chap. III).