

## Week 5 (due Nov. 1)

1. The transition amplitude in nonrelativistic Quantum Mechanics is defined by

$$K(q', q; T) = \langle q' | e^{-iHT} | q \rangle.$$

Here  $H$  is the usual Hamiltonian, i.e.

$$H = \frac{\hat{p}^2}{2m} + V(q).$$

(a) Compute  $K(q', q; T)$  for the free particle ( $V = 0$ ) by inserting identity operators in the form

$$1 = \int dp |p\rangle \langle p|,$$

and then evaluating the resulting matrix elements and integrals over  $p$ .

(b) On the other hand, one can consider the second-quantized version of the same system and the corresponding 2-point Green's function

$$G(q', q; T) = \langle 0 | \Psi(T, q') \Psi^\dagger(0, q) | 0 \rangle$$

Show that for any potential  $V(q)$  one has  $G(q', q; T) = K(q', q; T)$ . Verify this in the special case  $V = 0$  by directly evaluating  $G(q', q; T)$  using the known Fourier-expansion of  $\Psi$  and  $\Psi^\dagger$  in terms of creation-annihilation operators and then comparing with the results of part (a).

(c) The path-integral representation for  $K(q', q; T)$  is

$$K(q', q; T) = \int Dq(t) \exp(iS),$$

where

$$S = \int dt \left( \frac{m}{2} \dot{q}^2 - V(q) \right).$$

In more detail, the path-integral is defined as the limit

$$\lim_{N \rightarrow \infty} F(\epsilon)^N \int dq_1 \dots dq_{N-1} \exp \left[ i\epsilon \sum_{i=0}^{N-1} \left( \frac{m}{2} (q_{i+1} - q_i)^2 / \epsilon^2 - V(q_i) \right) \right],$$

where  $\epsilon = T/N$ ,  $q_0 = q$ ,  $q_N = q'$ , and the function  $F(\epsilon)$  should be chosen so that in the limit  $N \rightarrow \infty$  one gets the correct expression for  $K(q', q; T)$ .

Determine  $F(\epsilon)$  by evaluating the integral in the special case  $V = 0$  and comparing with the results of part (a).

(d) Consider now the case  $V(q) = -fq$ . This corresponds to a particle which is acted upon by a constant force  $f$ . Find  $K(q', q; T)$  by evaluating the path-integral and using the function  $F(\epsilon)$  found in part (c).