Week 5 (due Feb. 12)

1. Problem 48.5 (ab) (20pts).
2. Problem 52.3 (abcdefg) (50pts).
3. Consider a theory of Dirac fermions with mass m interacting via a quartic interaction

$$\mathcal{L}_{int} = g\bar{\psi}\gamma^{\mu}\psi\bar{\psi}\gamma_{\mu}\psi.$$

It describes spin-1/2 particles and anti-particles. In the nonrelativistic limit, one may write an effective interaction Hamiltonian for the Fermi-field describing particles alone. Determine this Hamiltonian to leading order in g and $1/m$ following the procedure outlined in class. Repeat the exercise for the interaction

$$\mathcal{L}'_{int} = g\bar{\psi}\gamma^{\mu}\gamma^{5}\psi\bar{\psi}\gamma_{\mu}\gamma^{5}\psi.$$

4. Consider a theory of a complex scalar field ϕ with mass M and self-interaction

$$\mathcal{L}_{int} = -\frac{\lambda}{4}(\phi^{\dagger}\phi)^2.$$

In the non-relativistic limit, this theory can be described by a pair of complex scalar fields Ψ_+ and Ψ_- describing particles and anti-particles respectively. Determine the effective nonrelativistic Hamiltonian describing their interactions, to leading order in g and $1/M$ expansion. (Hint: the simplest approach is to compute all 2-2 scattering amplitudes and match them to Born-approximation amplitudes in the nonrelativistic theory).