Week 3 (due Oct. 23)

1. (a) Consider free complex scalar field ϕ with mass m. The expansion of the field in terms of creation and annihilation operators is given in eq. (3.38) in Srednicky (Problem 3.5). Invert this formula and express $a_k, b_k, a_k^\dagger$ and $b_k^\dagger$ in terms of ϕ and $\phi^\dagger$.

(b) In an interacting theory we can define asymptotic multi-particle states using the same expressions for $a, a^\dagger, b, b^\dagger$ as in the free theory but evaluated at time $t = \pm\infty$. Following the same line of reasoning as in the real case, one can then relate the S-matrix elements with time-ordered Green's functions involving ϕ and $\phi^\dagger$. Use these considerations to formulate the LSZ theorem for the complex scalar field.

2. Consider the 2-point time-ordered Green's function for the free real scalar field. Show that it satisfies

$$(-\partial_\mu \partial^\mu + m^2) G_2(x, y) = -i\delta^4(x - y)$$

without using the explicit expression for $G_2(x, y)$ as a Fourier-integral. Rather, use the fact that $\phi(x)$ satisfies the Klein-Gordon equation, plus the equal-time canonical commutation relations

$$[\phi(t, \vec{x}), \phi(t, \vec{y})] = 0, \quad [\partial_0 \phi(t, \vec{x}), \partial_0 \phi(t, \vec{y})] = 0, \quad [\phi(t, \vec{x}), \partial_0 \phi(t, \vec{y})] = i\delta^3(\vec{x} - \vec{y}).$$